POLITEKNIK MALAYSIA

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# SHEAR FORCE AND BENDING MOMENT Fundamentals and Problem Solving

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**CIVIL ENGINEERING DEPARTMENT**

# SHEAR FORCE AND BENDING MOMENT

## Fundamentals and Problem Solving


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SHEAR FORCE AND BENDING MOMENT

Fundamentals and Problem Solving

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# PREFACE

The purpose of writing this eBook is to provide the students with a clear and thorough understanding of the basic principles and applications of shear force and bending moment in engineering structures. This eBook presents the principles behind the methods of solving  problem for determinate beam when subjected to different types of loads.

With these responsibilities in mind, the objective for this eBook is to develop the student's ability to recognize basic knowledge of engineering structures. We provide an adequate number of self-test and problem solving to enhance student knowledge and understanding.

Any suggestions, comments and feedback for further improvement are most welcome.

Herliana binti Hassan
Yusnita binti Yusof

# ACKNOWLEDGEMENTS

We would like to acknowledge the assistance and encouragement of our families and friends who have actively contributed either directly or indirectly to the completion of this eBook We are very grateful to the Head of Civil Engineering Department and colleagues for the encouragement and support and giving us the opportunity to produce this eBook.

We hope that this eBook can benefit students in increasing their understanding of the basic principles of mechanics of civil engineering structures

Herliana binti Hassan
Yusnita binti Yusof

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# 1.0 EQUILIBRIUM FORCE

## Learning Objective

Explain the principles and concepts of equilibrium forces.

Calculate values and directions of vertical reaction, horizontal reaction and moment.

## 1.1 PRINCIPLES OF EQUILIBRIUM FORCES

Engineering structures must remain in equilibrium both externally and internally when subjected to a system of forces. The principle of Equilibrium Forces states that if a structure is in equilibrium, all its members are also in equilibrium.

a. Summation of all forces is equal to zero

$$\Sigma f_x = 0$$

$$\Sigma f_y = 0$$

b. Summation of all moments in the structure is equal to zero

$$\Sigma M = 0$$

**PRINCIPLE 1**

Total leftwards forces is equals to the total of rightwards forces.

**PRINCIPLE 2**

Total upwards forces is equals to the total of downwards forces.

**PRINCIPLE 3**

Total clockwise moments is equals to the total of anticlockwise moments.

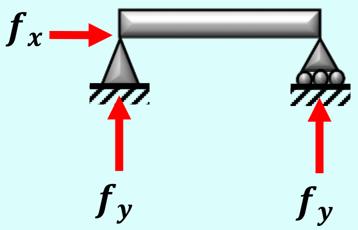
## 1.2 STATICALLY DETERMINATE AND INDETERMINATE BEAM

Classification of beams according to the equilibrium condition is categorized as:

**Statically determinate beam**

Beams that have three unknown support reactions and can be analyzed by using basic equilibrium conditions.

Example:

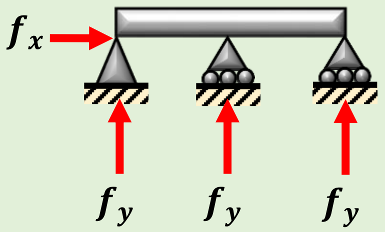



Three unknowns

**Statically indeterminate beam**

Beams that have more than three unknown support reactions and moments and cannot be analyzed by using basic equilibrium conditions.

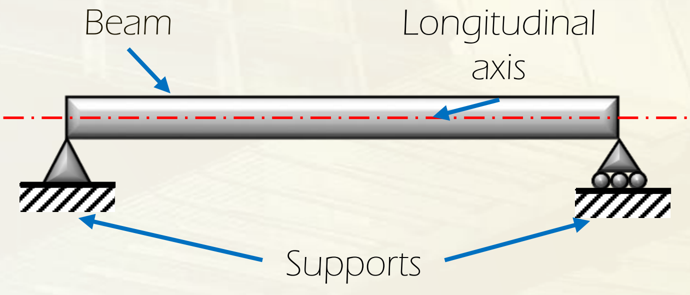
Example:




Four unknowns

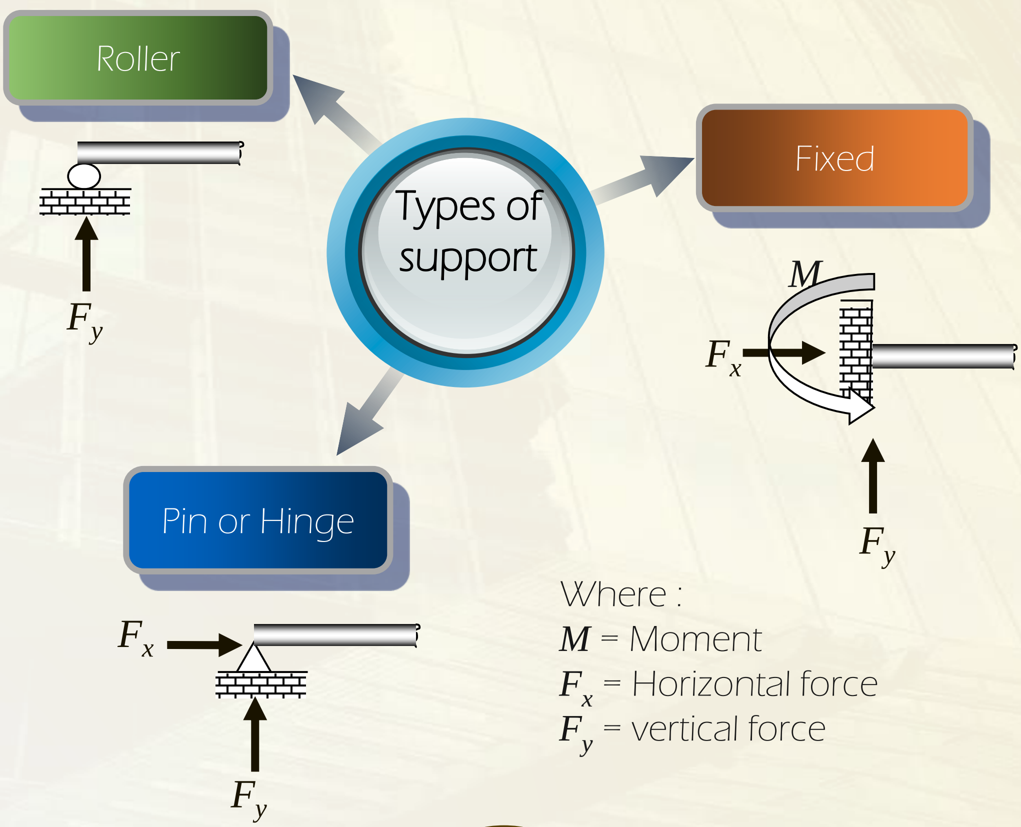
## 1.3 BEAM

Beam can be defined as a horizontal structural member spanning a distance between one or more supports and carrying vertical loads across its longitudinal axis. It is constructed to support the slab and withstand the loads coming from the building.



## 1.4 TYPES OF SUPPORT

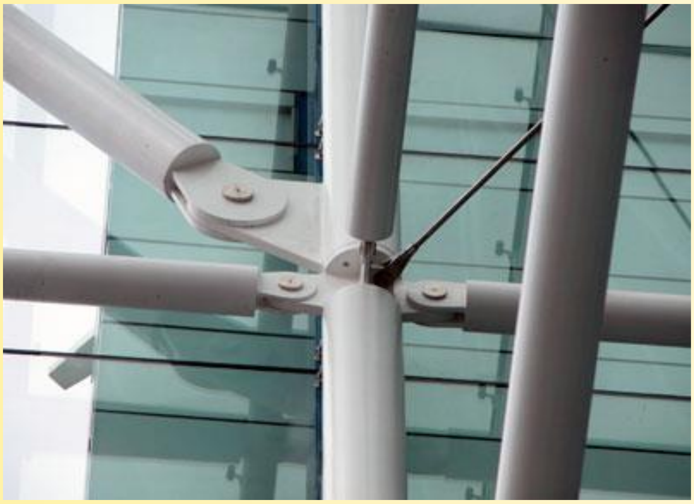
| Type of support | Total of unknown | Description |
|---|---|---|
| Roller support | 1 unknown<br>$\Sigma F_y = 0$ | A support condition that resists movement perpendicular to the point of contact. This support develops one unknown force component .Usually it reacts vertically. |
| Hinge or pin support | 2 unknowns<br>$\Sigma F_x = 0$<br>$\Sigma F_y = 0$ | A support condition that that can resist forces and prevent movement both in horizontally and vertically, but it does not prevent rotations. |
| Fixed Support | 3 unknowns<br>$\Sigma F_x = 0$<br>$\Sigma F_y = 0$<br>$\Sigma M = 0$ | A support condition that does not permit movement or rotation of the joining elements in any direction. This support develops three unknown reactions, including a vertical and horizontal component and a resisting moment. |



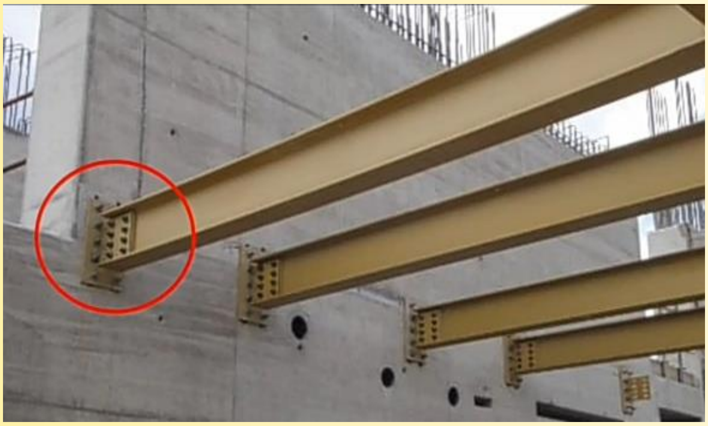

Where :
$M$ = Moment
$F_x$ = Horizontal force
$F_y$ = vertical force

## How does a support of beam look in real life?

Here are the examples application of a support in structures



Simple pin connections using two plates



Fixed Support - Beam Fixed in Wall

Source: Terri Meyer, B., & Vincent, H. (2020)

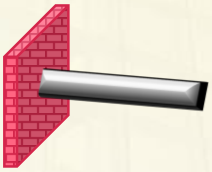
## 1.5 TYPE OF BEAMS

A beam is placed on supports and it is classified according to the support conditions as follows.

1. Simply Supported Beam

A beam in which one end is supported by a hinge/pin and the other end is supported by a roller.

2. Cantilever Beam



A beam that is fixed or anchored at one end and free at the other end.

3. Overhanging Beam

A beam that has one or both of its ends extending beyond the supports.

4. Continuous Beam

A beam that spans over more than two supports.

Simply supported beam, cantilever beam and overhanging beam are categorized as Statically Determinate Beam

Continuous beam is categorized as Statically Indeterminate Beam

## 1.6 MAIN TYPES OF EXTERNAL LOAD

Structures are designed to withstand various types of loads.

### DEAD LOAD

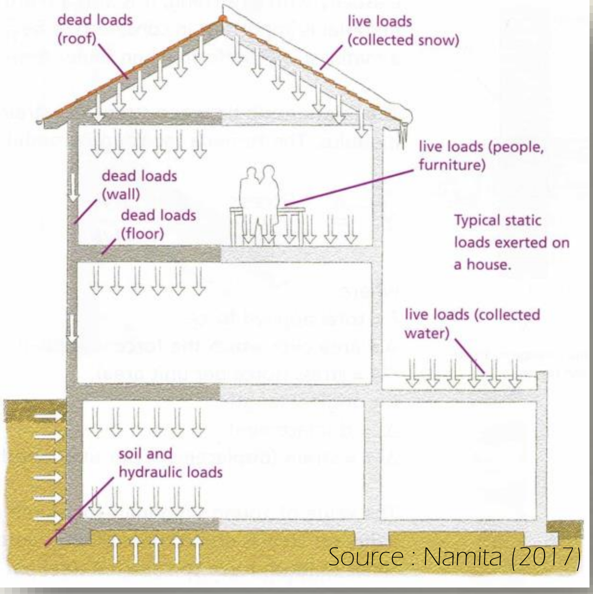
Dead loads are due to self weight of the structure. Dead loads are the permanent loads which are always present, and it depends upon the unit weight of the material. Example Self weight of walls, floors, beams, columns etc.

### LIVE LOAD

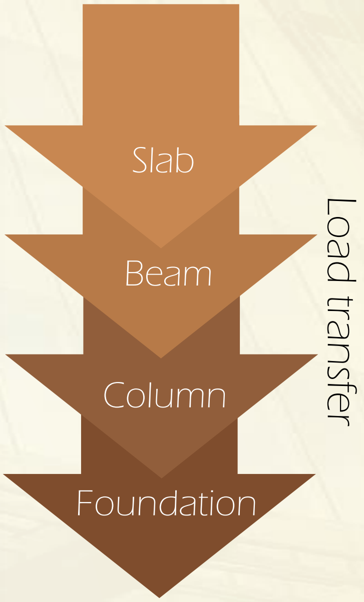
Live loads (also called as imposed loads) on floors and roofs consists of all the loads which are temporarily placed on the structure. Live loads keep on changing from time to time and it depend upon the use of building. Example, loads of people, furniture, machines etc.

### WIND LOAD

The force exerted by the horizontal component of wind is to be considered in the design of building. Wind loads depends upon the velocity of wind, shape and size of the building.




General classification of loads

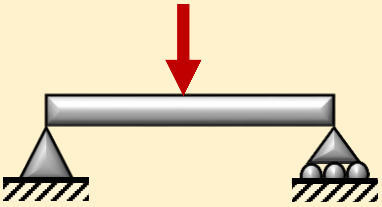


## 1.7 LOAD DISTRIBUTION

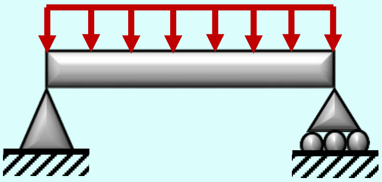
There are three main methods how loads are distributed on a structure. However, loads distributed in a structure are the combination of many types of loads.

### Point Load

- ✓ This load acts on a very small area and can be considered as acting on a point.
- ✓ Its symbol is an arrow, and its unit is N or kN



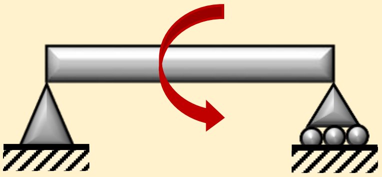

### Uniformly Distributed Load (UDL)




- ✓ This type of loads acts along the whole length of a beam or some part of it and the magnitude of the load is equal.
- ✓ Its unit is N/m or kN/m.

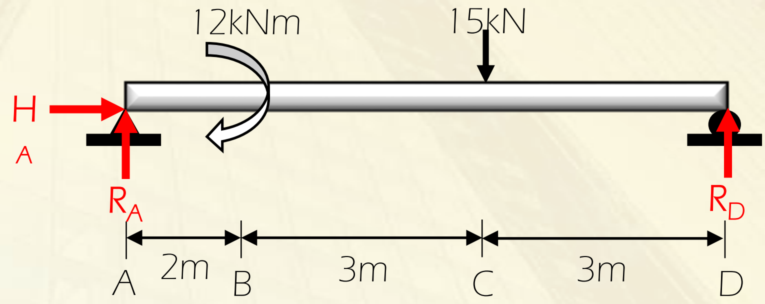
### Moment

- ✓ Moment is generated by a pair of force.
- ✓ These forces are acting on a point and this results in a twist at the point.
- ✓ The direction of the twist is either clockwise or anticlockwise.
- ✓ Its unit is Nm or kNm.



# 1.8 FREE BODY DIAGRAM

Any force system acting on a structure is easily analyzed if the appropriate reaction required to maintain equilibrium are inserted in a diagram is called *Free Body Diagram*.




Digital Link

https://youtu.be/C-FEVzl8oe8

## Example 1.1 Simply Supported Beam

Based on figure 1.1, determine reactions at the support of the simply supported beam.

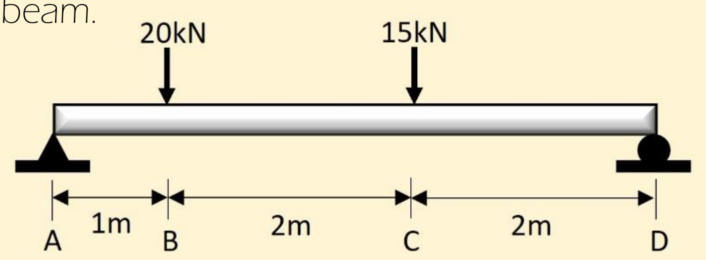
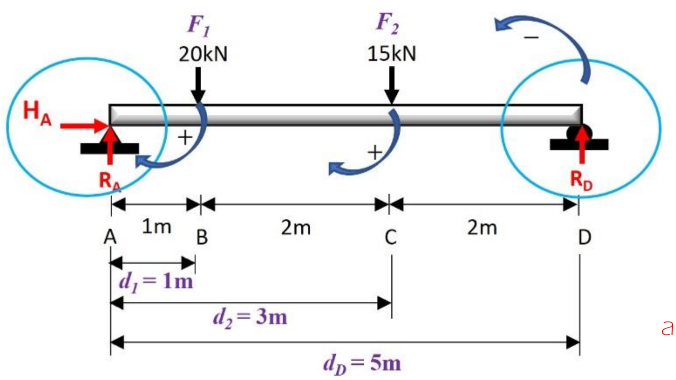



Figure 1.1

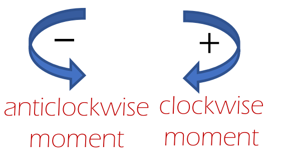
Solution:

Step 1: Sketch Free Body Diagram.

Pin Support :
- 2 unknowns
- $f_y$ and $f_x$




Roller Support :
- 1 unknown
- $f_y$




Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at A, $H_A$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A = 0$$

Use summation of Moment at point A to get the vertical reaction at D, $R_D$

$$\Sigma M_A = 0;$$

$$F_1 d_1 + F_2 d_2 - R_D d_D = 0$$

$$(20kN)(1m) + (15kN)(3m) - (R_D)(5m) = 0$$

$$R_D = 13\ kN$$

Use summation of Vertical forces to get the vertical reaction at A, $R_A$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A + R_D = 20kN + 15kN$$

$$R_A + 13 = 35kN$$

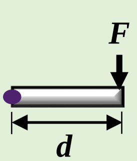
Substitute $R_D = 13\ kN$

$$R_A = 22kN$$

### Things to keep in mind

Moment = Force × distance

$\boldsymbol{M} = \boldsymbol{F} \times \boldsymbol{d}$




**Force is perpendicular to the distance where moment is to be taken

## Example 1.2 Simply Supported Beam

Based on figure 1.2, determine reactions at the support of the simply supported b

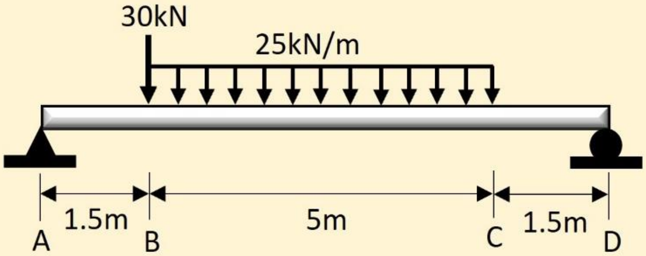
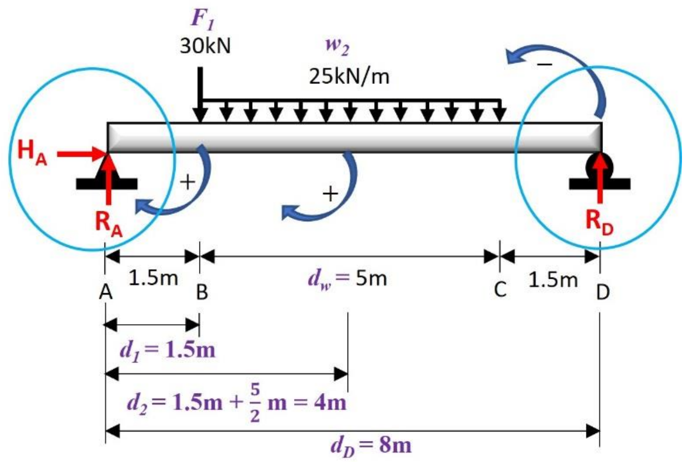



Figure 1.2

Solution:

Step 1: Sketch Free Body Diagram.

Pin Support :
- 2 unknowns
- $f_y$ and $f_x$




Roller Support :
- 1 unknown
- $f_y$

Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at A, $H_A$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A = 0$$

Use summation of Moment at point A to get the vertical reaction at D, $R_D$

$$\Sigma M_A = 0;$$

$$F_1 d_1 + w_2 d_w d_2 - R_D d_D = 0$$

$$(30kN)(1.5m) + (25\frac{kN}{m})(5m)(4m) - (R_D)(8m) = 0$$

$$R_D = 68.125kN$$

Use summation of Vertical forces to get the vertical reaction at A, $R_A$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A + R_D = 30kN + (25\frac{kN}{m})(5m)$$

$$R_A + 68.125 = 155kN$$

Substitute $R_D = 68.125\ kN$

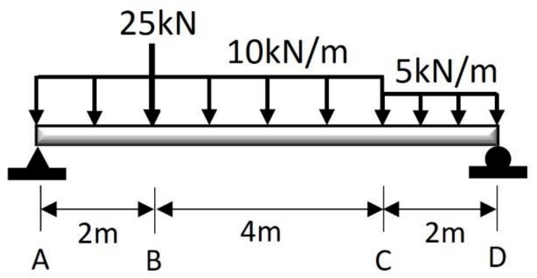
$$R_A = 86.875kN$$

# SELF-TEST 1A

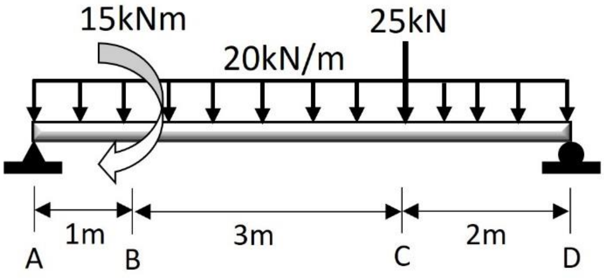


Determine reactions at the support for each of the simply supported beam as shown in figure below.

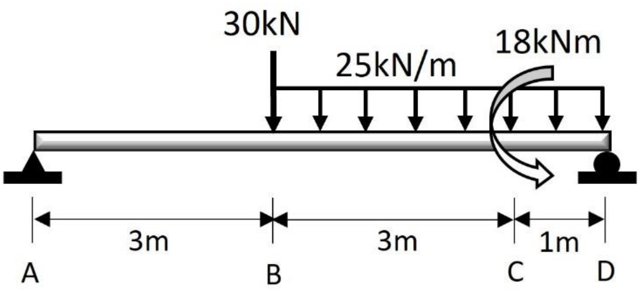
1.



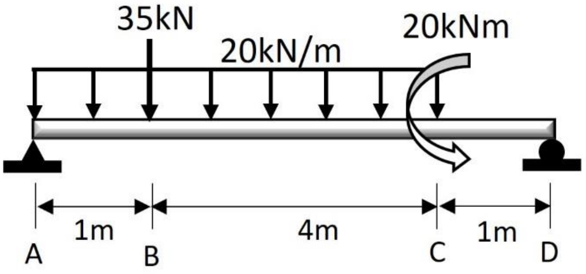

2.




3.




4.



## Example 1.3 Cantilever Beam

Based on figure 1.3, determine reactions at the support of the cantilever beam.

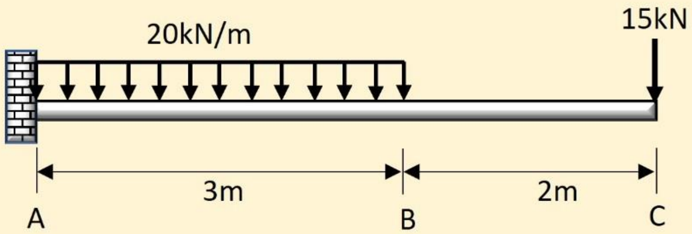


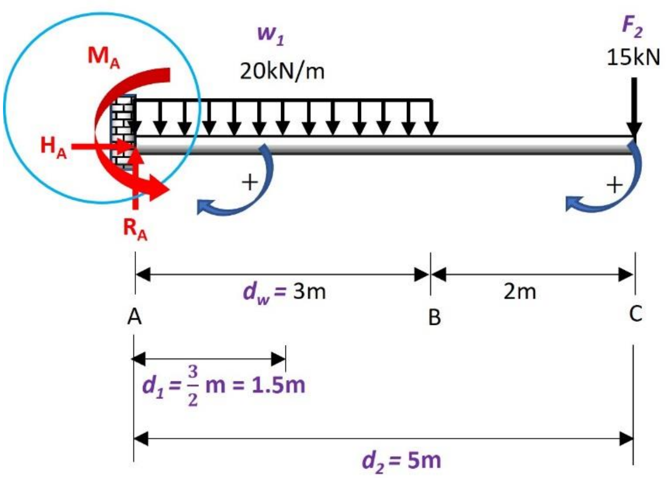

Figure 1.3

Solution:

Step 1: Sketch Free Body Diagram.

Fixed Support :

- 3 unknowns
- $f_y$, $f_x$ and M




Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at A, $H_A$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A = 0$$

Use summation of Vertical forces to get the vertical reaction at A, $R_A$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A = \left(20\frac{kN}{m}\right)(3m) + 15kN$$

$w_1 d_w$

$$R_A = 75kN$$

Use summation of Moment at point D to get the Moment at A, $M_A$

$$\Sigma M_A = 0;$$

$$w_1 d_w d_1 + F_2 d_2 - M_A = 0$$

$$\left(20\frac{kN}{m}\right)(3m)(1.5m) + (15kN)(5m) - M_A = 0$$

$$M_A = 165kNm$$

## Example 1.4 Cantilever Beam

Based on figure 1.4, determine reactions at the support of the cantilever beam.

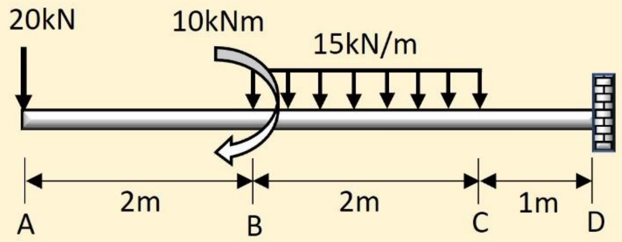
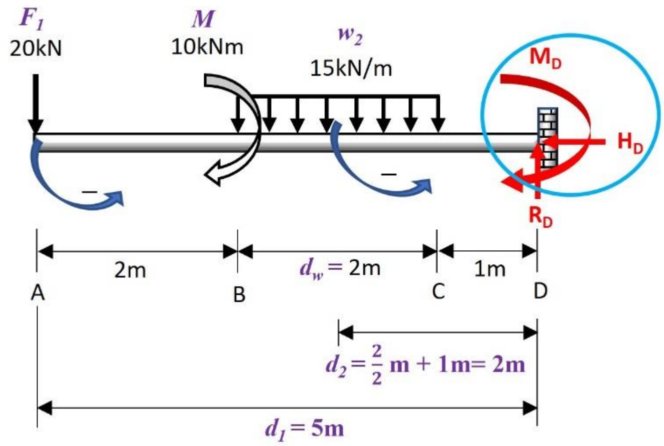



Figure 1.4

Solution:

Step 1: Sketch Free Body Diagram.




Fixed Support :

- 3 unknowns
- $f_y$, $f_x$ and M

Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at D, $H_D$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_D = 0$$

Use summation of Vertical forces to get the vertical reaction at D, $R_D$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_D = (20kN) + \left(15\frac{kN}{m}\right)(2m)$$

$w_2 d_w$

$$R_D = 50kN$$

Use summation of Moment at point D to get the Moment at D, $M_D$

$$\Sigma M_D = 0;$$

$$-F_2 d_2 + M - w_2 d_w d_2 - M_D = 0$$

$$-(20kN)(5m) + 10kNm - \left(15\frac{kN}{m}\right)(2m)(2m) + M_D = 0$$

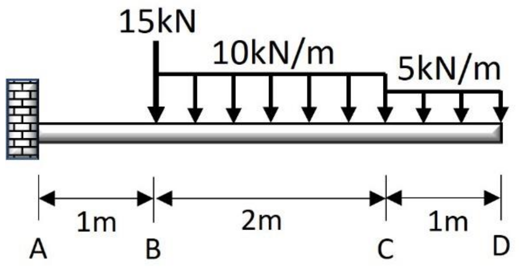
$$M_D = 150kNm$$

# SELF-TEST 1B

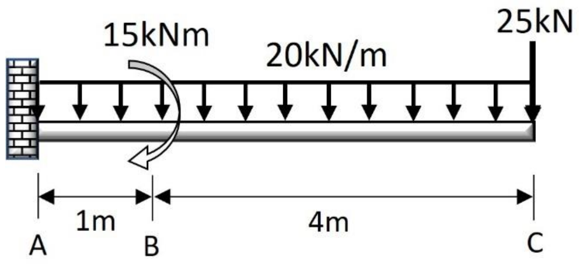


Determine reactions at the support of cantilever beam as shown in figure below.

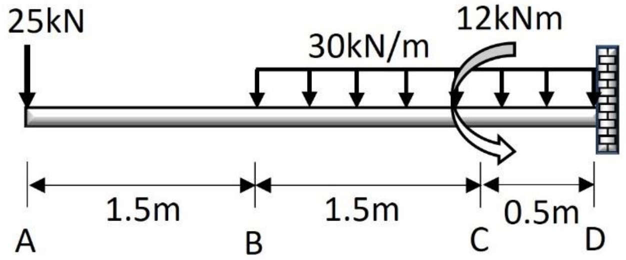
1.



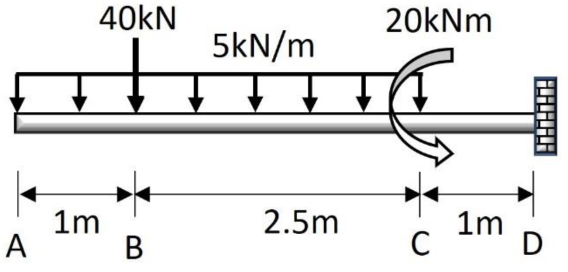

2.




3.




4.



## Example 1.5 Cantilever Beam

Based on figure 1.5, determine reactions at the support of the overhanging beam.

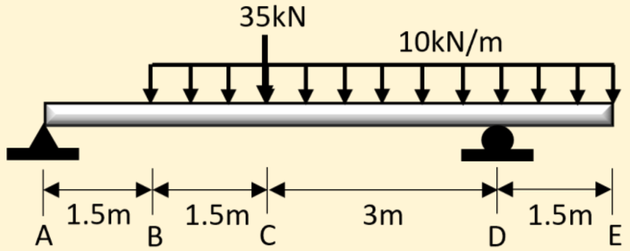
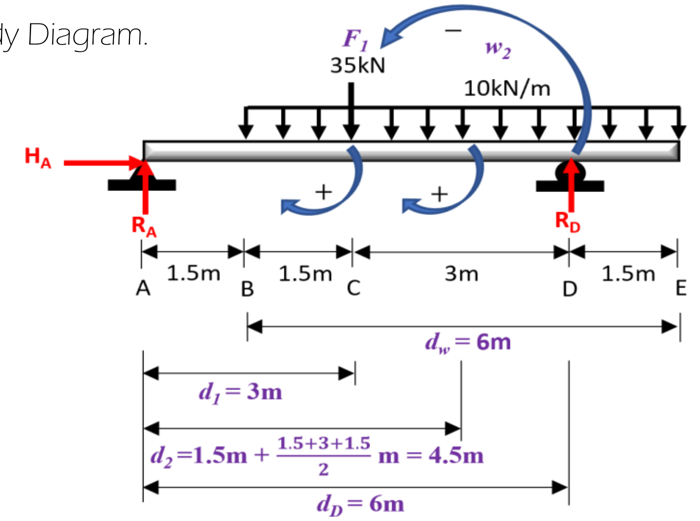



Figure 1.5

Solution:

Step 1: Sketch Free Body Diagram.




Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at A, $H_A$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A = 0$$

Use summation of Moment at point A to get the vertical reaction at D, $R_D$

$$\Sigma M_A = 0;$$

$$F_1 d_1 + w_2 d_w d_2 - R_D d_D = 0$$

$$(35kN)(3m) + (10\frac{kN}{m})(6m)(4.5m) - (R_D)(6m) = 0$$

$$R_D = 62.5kN$$

Use summation of Vertical forces to get the vertical reaction at A, $R_A$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A + R_D = 35kN + (10\frac{kN}{m})(6m)$$

$$R_A + 62.5 = 95kN$$

Substitute $R_D = 62.5\ kN$

$$R_A = 32.5kN$$

## Example 1.6 Cantilever Beam

Based on figure 1.6, determine reactions at the support of the overhanging beam.

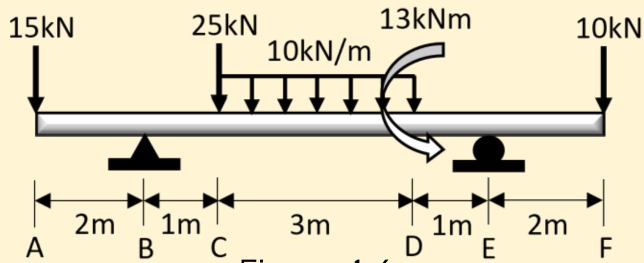
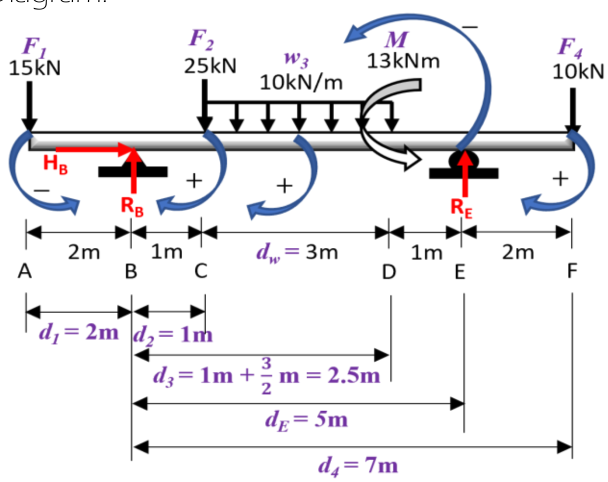



Figure 1.6

Solution:

Step 1: Sketch Free Body Diagram.




Step 2: Determine reaction at support D by using principle of equilibrium forces.

Use a summation of Horizontal forces to get the horizontal reaction at B, $H_B$

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_B = 0$$

Use summation of Moment at point B to get the vertical reaction at E, $R_E$

$$\Sigma M_B = 0;$$

$$-F_1 d_1 + F_2\, d_2 + w_3 d_w d_3 - M - R_E d_E + F_4\, d_4 = 0$$

$$-(15kN)(2m) + (25kN)(1m) + \left(10\frac{kN}{m}\right)(3m)(2.5m) - 13 - (R_E)(5m) + (10kN)(7m) = 0$$

$$R_E = 25.4kN$$

Use summation of Vertical forces to get the vertical reaction at B, $R_B$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_B + R_E = 15kN + 25kN + \left(10\frac{kN}{m}\right)(3m) + 10kN$$

$$R_B + 11 = 80kN$$

Substitute $R_E = 25.4\ kN$

$$R_B = 54.6kN$$

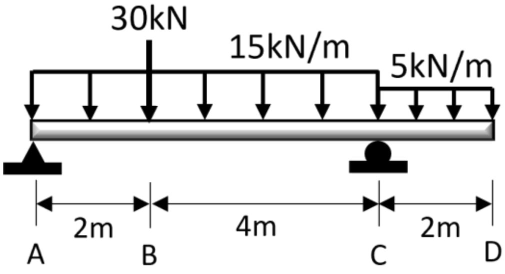
# SELF-TEST 1C

## Overhanging Beam

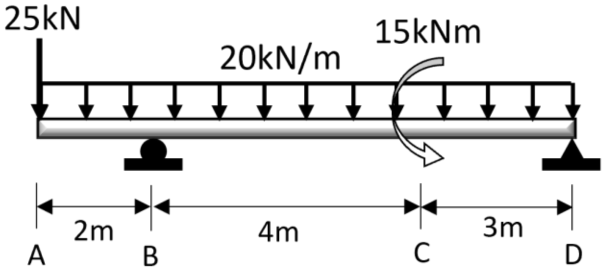


Determine reactions at the support for each of the overhanging beam as shown in figure below.

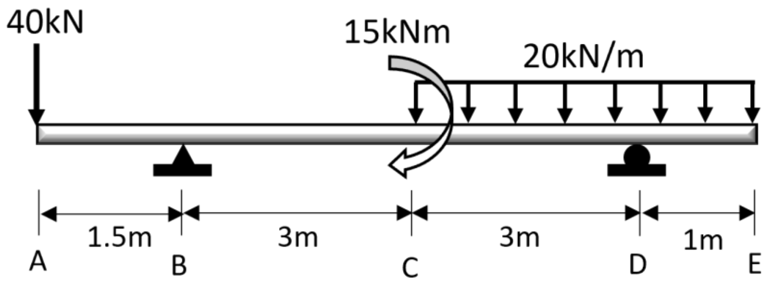
1.



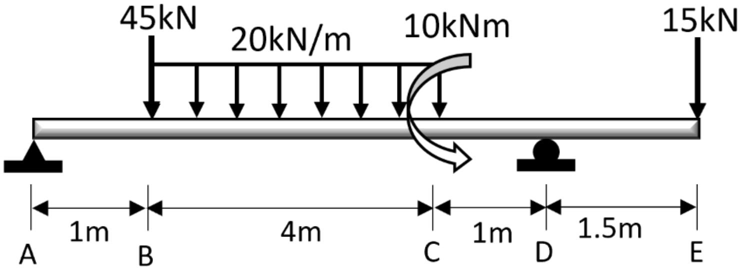

2.




3.




4.



# 2.0 SHEAR FORCE AND BENDING MOMENT

## Learning Objective

Calculate shear force and bending moment in beam.

Draw shear force and bending moment diagrams for statically determinate beams.

## 2.1 WHAT IS SHEAR FORCE AND BENDING MOMENT?

### Shear Force

Shear forces are internal forces developed in the beam material to balance the externally applied forces in order to secure the equilibrium of all members of the beam.

### Bending Moment

Bending moments are internal moments that are developed in the beam material to balance the tendency of external forces to cause rotation of any part of the beam.

Shear force diagram and bending moment diagram is the most important first step toward design calculations of structural members.

Shear Force diagram indicates the shear force resisted by the beam section along the length of that beam.

Bending Moment Diagram is the diagram to show how the applied loads to a beam create a moment variation along the length of the beam.

## Why do we need to calculate Shear Force and Bending Moment?

1. As analytical tools used in conjunction with structural analysis to help perform structural design by determining the value of shear force and bending moment at a given point of a structural element such as a beam.

2. These diagrams can be used to provide an appropriate size of beam or column which can resist the applied forces when a given set of loads, it can be supported without structural failure.

Digital Link

https://youtu.be/5K27dJqGpf8

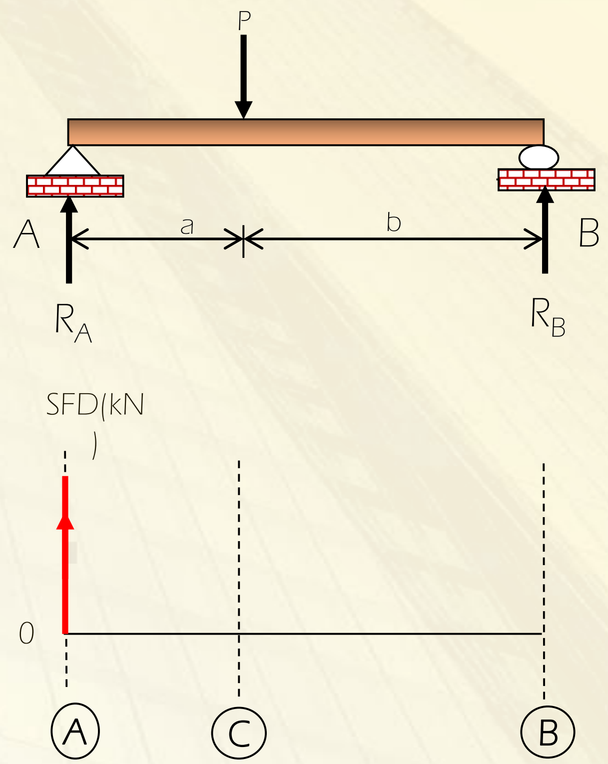
## 2.2 RULES TO CONSTRUCT SHEAR FORCE AND BENDING MOMENT DIAGRAM

2.0

**Rule 1** Start with the far-LEFT SIDE of the beam.

If there is an upward force at a support, then the Shear force diagram will start at this force above the *x*-axis.

If there is a downward point load and no support, then the shear force diagram will start as a negative at the value of the point load

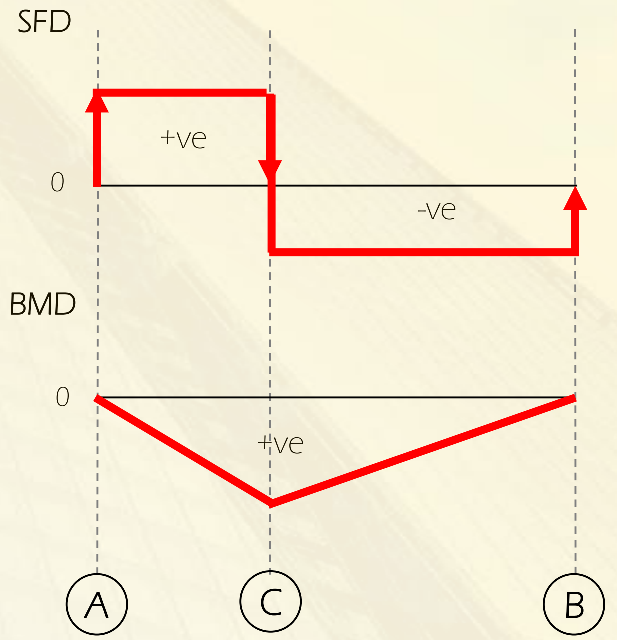



**Rule 2** Move across the beam to right side, stop at every load that acts on the beam

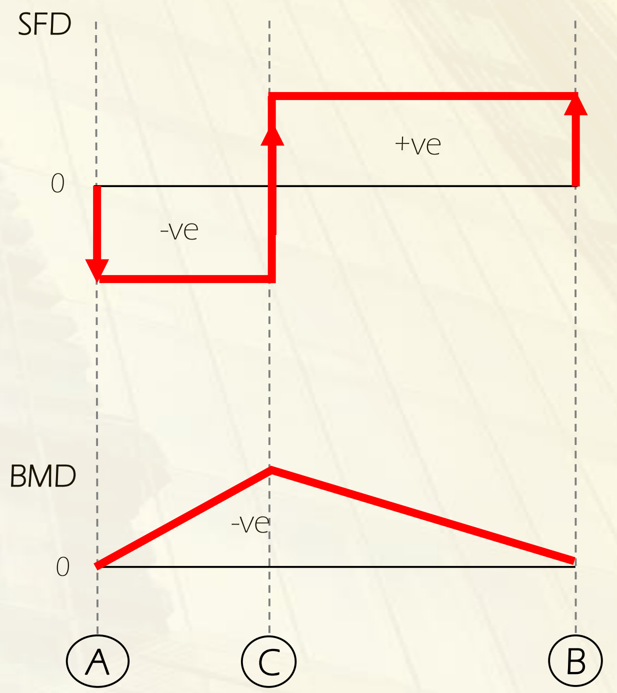
When come across the loads, simply add (or subtract) these loads from the value you already have (keeping a *cumulative total*)

**Rule 3** Draw shear force and bending moment diagram by following the sign convention as below:

POSITIVE Shear Force Diagram is ABOVE the ***x-x*** axis while NEGATIVE Shear Force Diagram is BELOW the ***x-x*** axis




POSITIVE Bending Moment Diagram is BELOW the ***x-x*** axis while NEGATIVE Bending Moment Diagram is ABOVE the ***x-x*** axis



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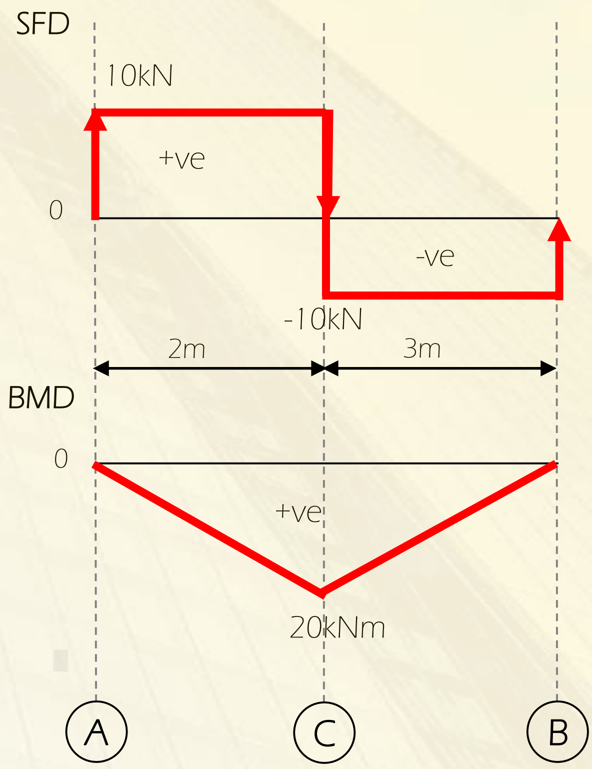
**Rule 4** Value of bending moment between any two point is equal to the AREA of the shear force diagram

When come across loads, simply add (or subtract) these loads from the value you already have, keeping a cumulative total




Consider a shear force diagram as shown in figure, therefore, bending moment at each point is calculated as follow:

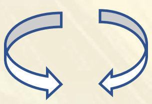
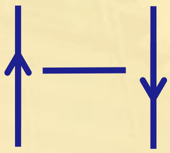
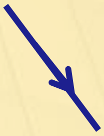
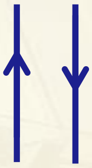
$M_A = 0$
$M_C = (10 \times 2) = 20$
$M_B = 20 - (10 \times 2) = 0$

# 2.3 VARIATION OF SHEAR FORCE AND BENDING MOMENTS

1. Where a beam experiences point loads:
   The shear force diagram comprises horizontal straight lines.
   The bending moment diagram is sloping straight lines.

2. Where a beam experiences uniformly distributed loads:
   The shear force diagram comprises sloping straight lines.
   The bending moment diagram is curved (parabolic).

3. Where a beam experiences moment:
   The shear force diagram comprises horizontal straight lines.
   The bending moment diagram is vertical line (downward or upward).

| Types of load | Point Load (kN)  | UDL (kN/m)  | Moment (kNm)  |
|---|---|---|---|
| Shear Force Diagram, SFD (kN) | Vertical & Horizontal straight lines  | Sloping straight lines  | Horizontal straight lines  |
| Bending Moment Diagram, BMD (kNm) | Sloping straight lines  | Curved line (parabolic)  | Vertical line  |

# 2.4 TYPICAL DIAGRAM OF SHEAR FORCE AND BENDING MOMENT FOR DETERMINATE BEAM

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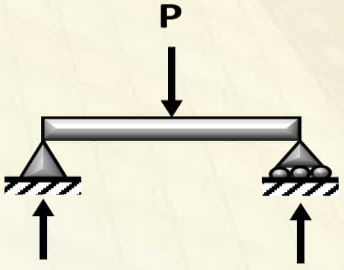
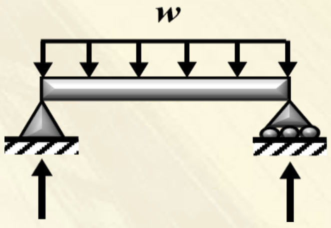
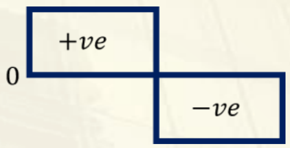
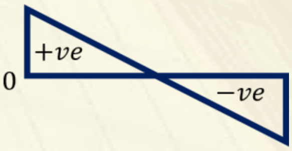
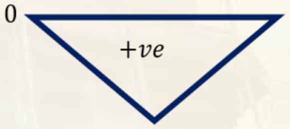
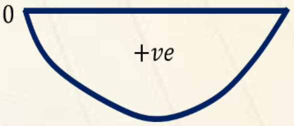
## IMPORTANT concepts of SIMPLY SUPPORTED BEAM

| Types of load | Simply supported beam subjected to point Load | Simply supported beam subjected to uniformly distributed Load |
|---|---|---|
| |  |  |
| Shear Force Diagram, SFD (kN) |  |  |
| Bending Moment Diagram, BMD (kNm) |  |  |

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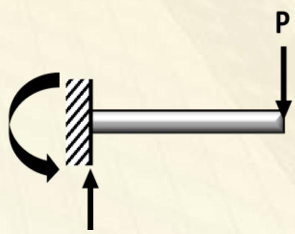
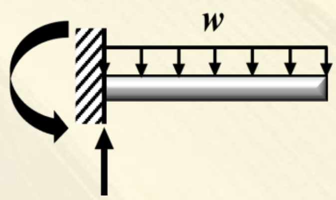
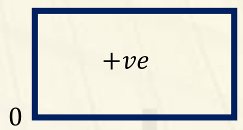
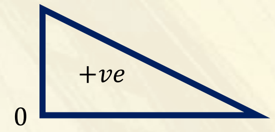
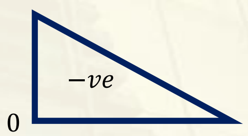
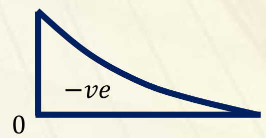
## IMPORTANT concepts of CANTILEVER BEAM

| Types of load | Cantilever beam subjected to point Load | Cantilever beam subjected to uniformly distributed Load |
|---|---|---|
| |   |   |
| Shear Force Diagram, SFD (kN) |   |   |
| Bending Moment Diagram, BMD (kNm) |   |   |

## Example 2.1 Simply Supported Beam

### CASE 1: Beam with a non-central point load

Construct shear force and bending moment diagrams for a beam as shown in Figure 2.1.

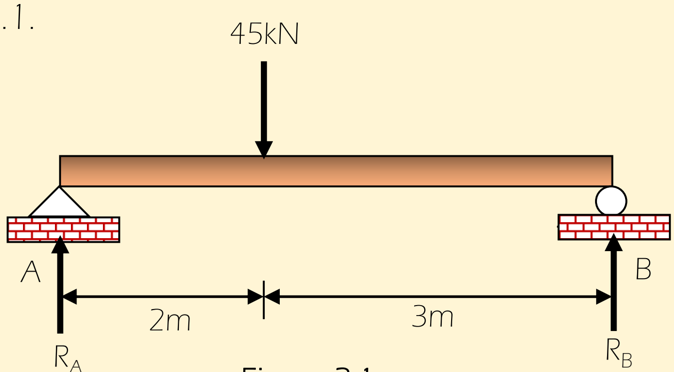
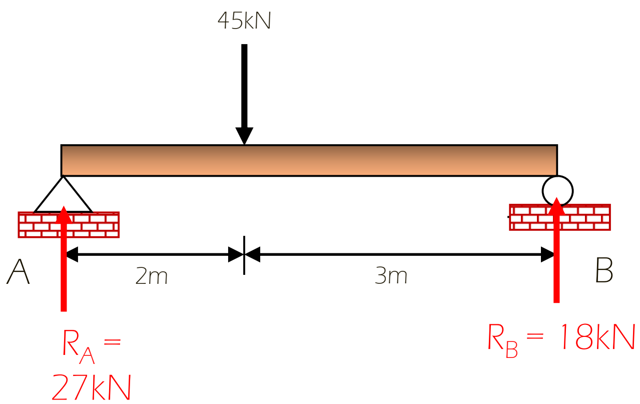
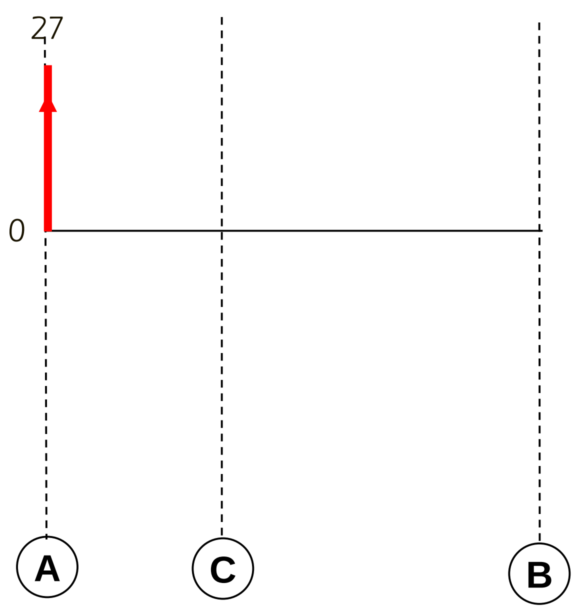



Figure 2.1

Solution:




SFD (kN)




Step 1: Calculate reactions at support A and B.

$$\Sigma M_A = 0$$
$$45(2) - R_B(5) = 0$$
$$R_B = 18\text{kN}$$

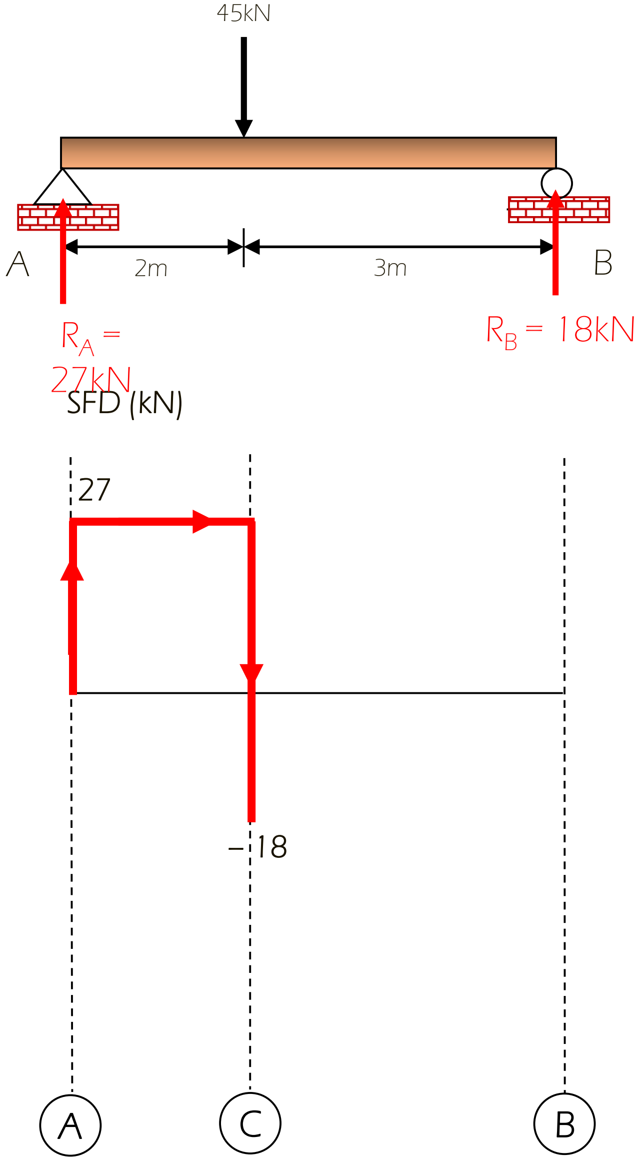
$$\Sigma F_y \uparrow = \Sigma F_y \downarrow$$
$$R_A + R_B = 45$$
$$R_A = 27kN$$

Step 2: Drop vertical lines at each load and draw horizontal lines on which to construct shear force diagrams (SFD).

Step 3: Draw SFD from the left side of the beam A and move across the beam. In this case it is a +27kN due to the reaction at point A because the reaction arrow is pointed up 27kN.

# Example 2.1 Simply Supported Beam






Step 4: Keep moving across the beam, stopping at every load that acts on the beam. The value on the shear diagram remains unchanged from point A to point C

There is a downward 45kN force at point C, so, minus 45kN from the existing 27kN.

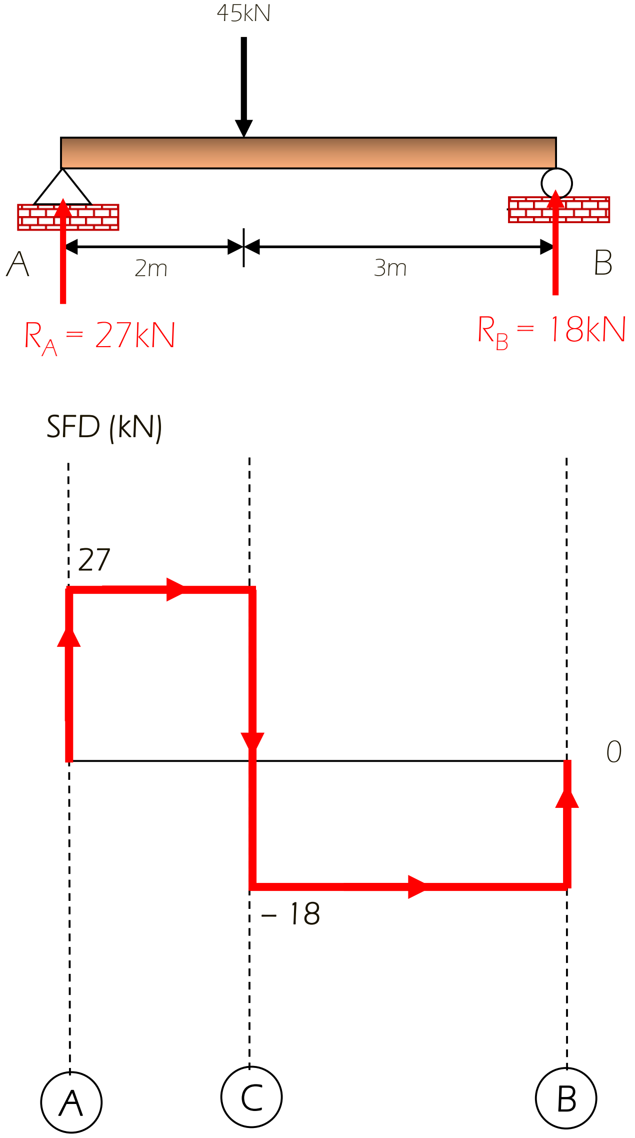
Therefore;

27kN – 45kN = – 18kN

which pushes the shear diagram down from its present value by 45kN.

## Example 2.1 Simply Supported Beam






Step 5: Moving across the beam again, a positive reaction of 18kN acting at support B.

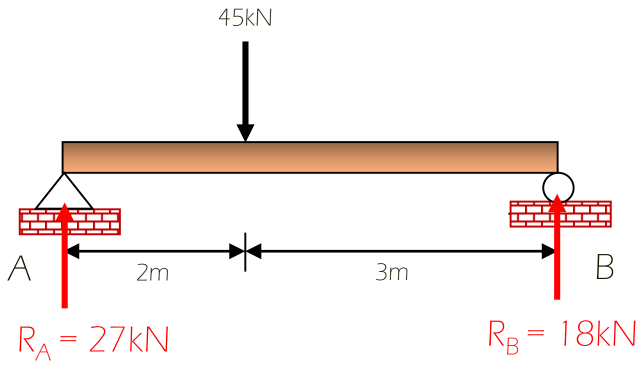
Again, add this +18kN to the shear force diagram (which is currently at –18kN) which will bring to a shear force of 0.

–18kN + 18kN = 0kN which pushes the shear diagram up to zero.

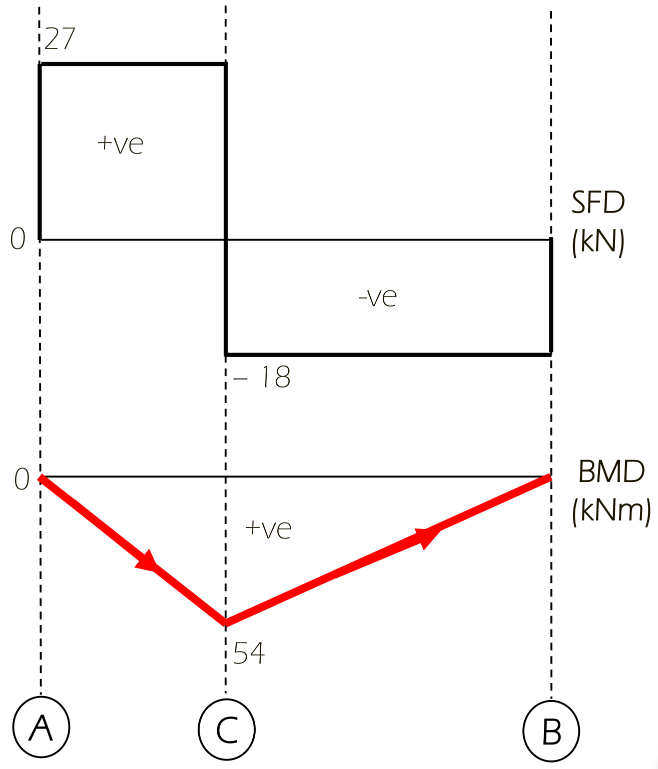
Since we are at the end of the beam, we will go no further, and we have our final Shear Force Diagram (SFD)

## Example 2.1 Simply Supported Beam






Step 6: Calculate bending moment at each point on the beam.




BMD (kNm)

$M_A = 0$

$M_C = (27 \times 2) = 54$

$M_B = 54 - (18 \times 3) = 0$

To construct Bending Moment Diagram, start at A where the moments at the support A is zero.

Note that the bending moment diagram comprises sloping straight line due to the point loaded between two points.

## Example 2.2 Simply Supported Beam

### CASE 2: Beam carrying a uniformly distributed load over its entire beam span

Construct shear force and bending moment diagrams for a beam as shown in Figure 2.2.

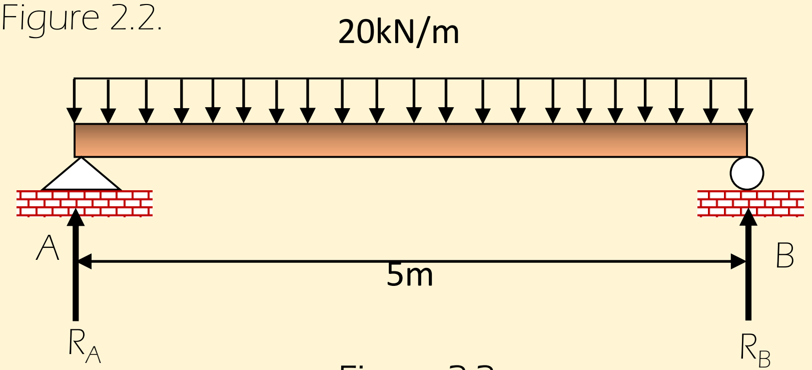
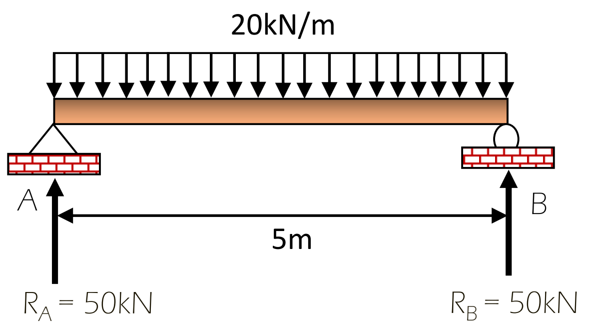


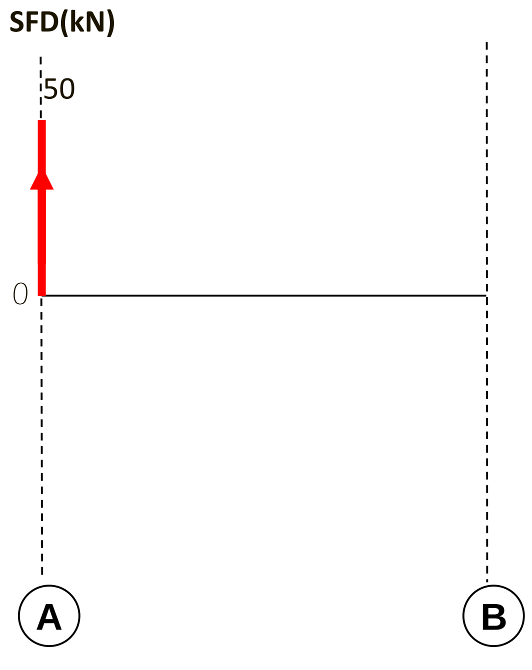

Figure 2.2

Solution:




Step 1: Calculate reactions at support A and B.

In this case, the reaction at A and B is 50kN.

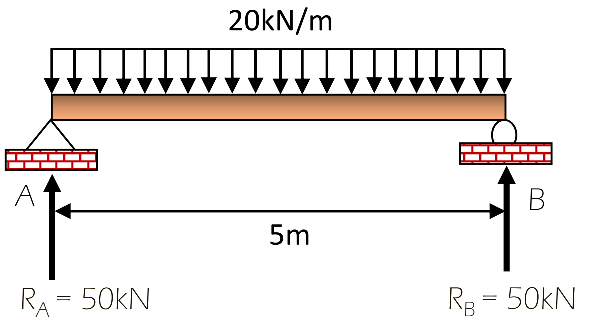


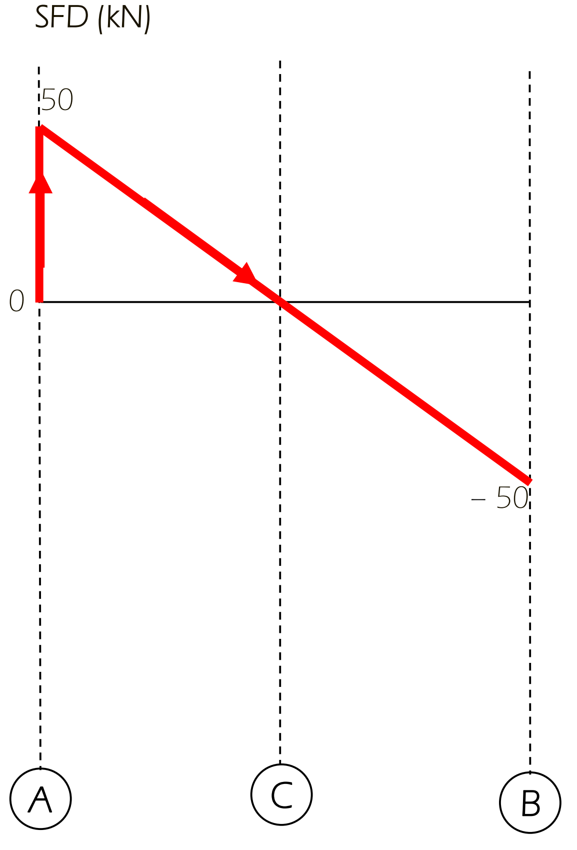

Step 2: Drop vertical lines at each load and draw horizontal lines on which to construct shear force diagrams.

Step 3: Draw SFD from the left side of the beam A and move across the beam. In this case it is a +50kN due to the reaction at point A because the reaction arrow is pointed up 50kN.

# Example 2.2 Simply Supported Beam





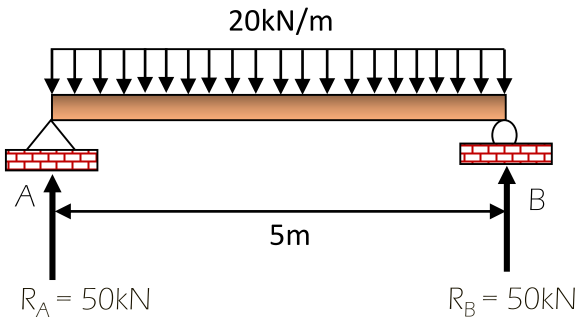

Step 4: Keep moving across the beam, stop at every load that acts on the beam.

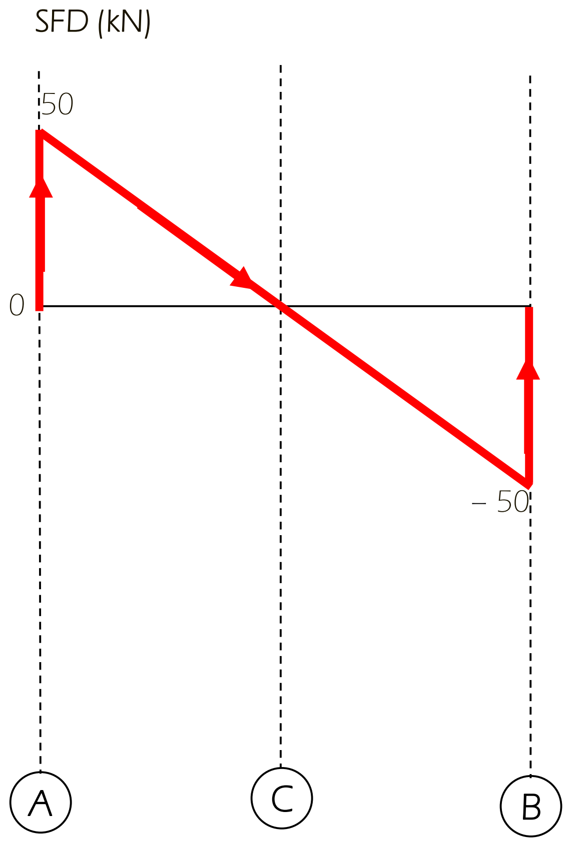
A uniform distributed load of 20kN/m along the span, so we will minus 20kN/m from the existing 50kN.

50kN – 20(5)kN = – 50kN which pushes the shear diagram down as sloping straight line from its present value by 20kN/m.

## Example 2.2 Simply Supported Beam






Moving across the beam again, a positive reaction of 50kN acting at support B.

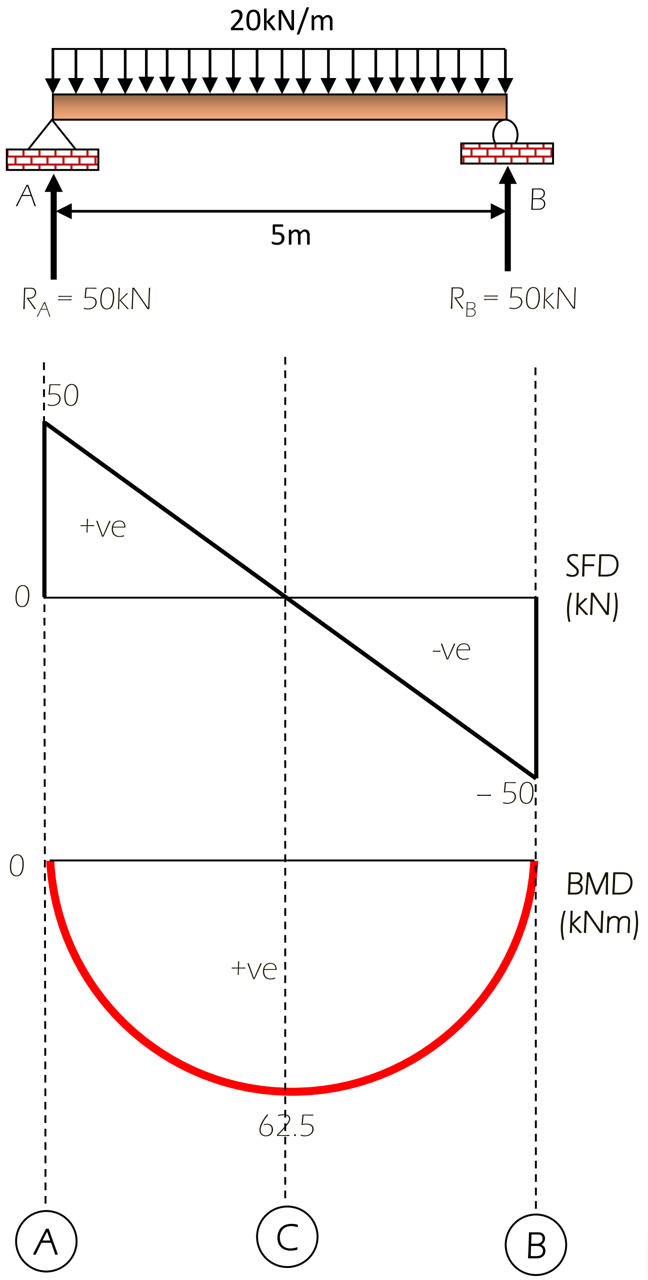
Again, add this +50kN to the shear force diagram (which is currently at -50kN) which will bring us to a shear force of 0.

– 50kN + 50 kN = 0 kN

which pushes the shear diagram up to zero.

Since we are at the end of the beam, we will go no further and we have our final Shear Force Diagram (SFD)

## Example 2.2 Simply Supported Beam




To construct Bending moment Diagram, start at A where the moments at the supports A is zero.

Then, move across the beam and calculate the bending moment at each point.

<u>BMD (kNm)</u>

$M_A = 0$

$M_C = (\frac{1}{2} \times 2.5 \times 50)$

$= 62.5$

$M_B = 62.5 - (\frac{1}{2} \times 2.5 \times 50)$

$= 0$

Note that the bending moment diagram comprises curved line due to the uniformly distributed load between two points.

## Example 2.3 Simply Supported Beam

A simply supported beam loaded as shown in Figure 2.3. If reaction at support A are $R_A$ = 22kN and $R_D$ = 13kN.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.



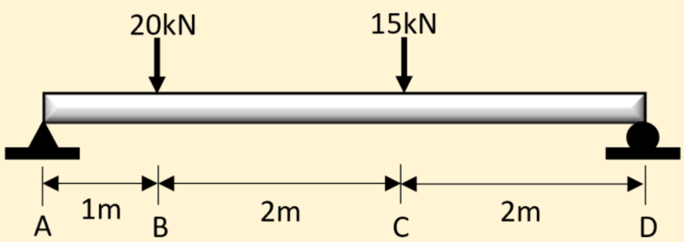
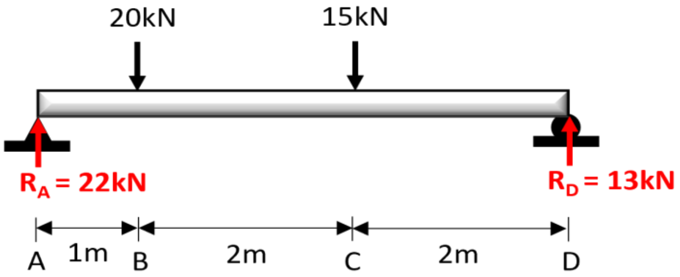
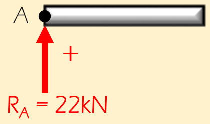
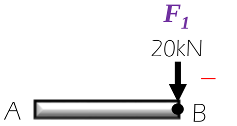
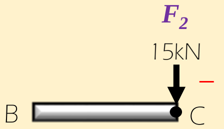
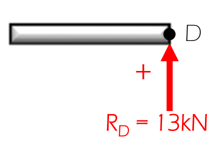



Figure 2.3

Solution:

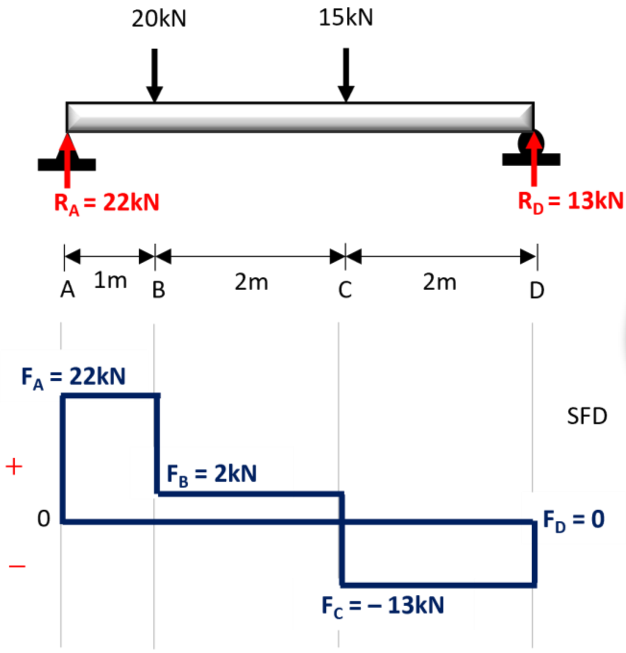


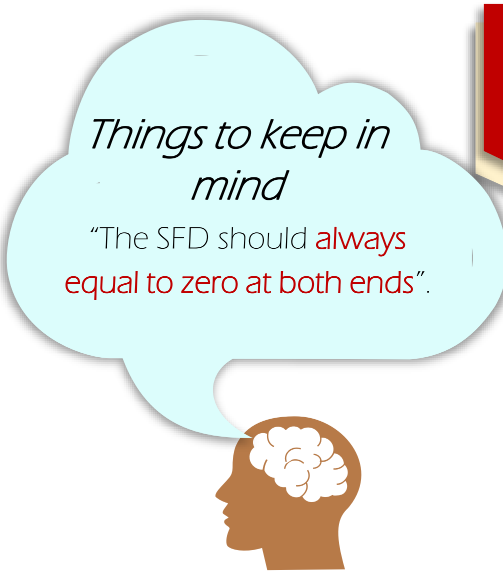

Step 1: Calculate Shear Force, kN.

| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |  | $F_A = R_A = +22kN$ |
| B |  | $F_B = F_A - F_1$<br>$F_B = 22kN - (20\text{kN}) = 2kN$ |
| C |  | $F_C = F_B - F_2$<br>$F_C = 2kN - 15kN = -13\text{kN}$ |
| D |  | $F_D = F_C - R_D$<br>$F_D = -13kN + 13kN = 0$ |

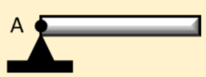
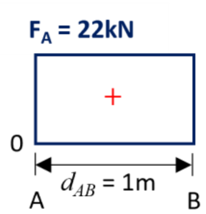
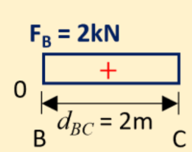
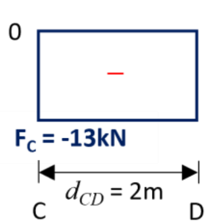
## Example 2.3 Simply Supported Beam

Step 2: Illustrate the Shear Force Diagram (SFD) .



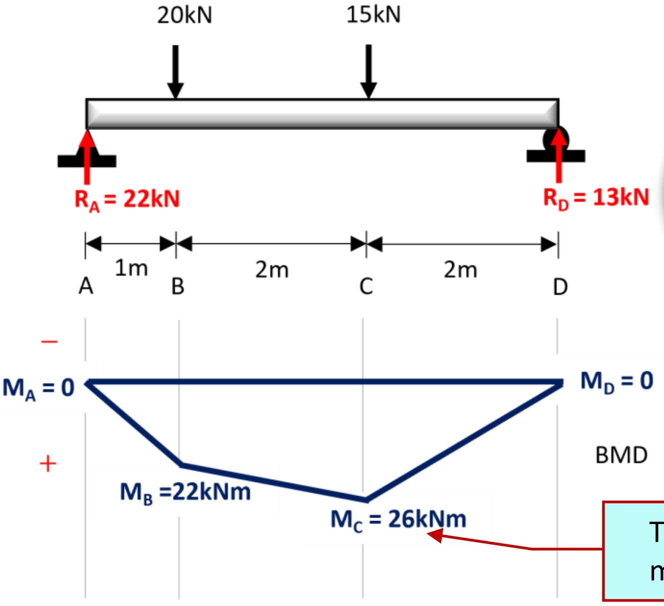




Step 3: Based on SFD and Figure 2.3, calculate Bending Moment, kNm.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$<br><br>TIP: Moment at support of a simply supported beam equal to zero. |
| A – B |   | Area of rectangle<br>$M_B = F_A d_{AB}$<br>$M_B = (22\text{kN})(1m)$<br>$M_B = 22\text{kNm}$ |
| B – C |   | $M_C = M_B + F_B d_{BC}$<br>$M_C = 22kN + (2kN)(2\text{m})$<br>$M_C = 26\text{kNm}$ |
| C – D |   | $M_D = M_C + F_C d_{CD}$<br>$M_D = 26kN + (-13kN)(2\text{m})$<br>$M_D = 0$ |

## Example 2.3 Simply Supported Beam

Step 4: Illustrate the Bending Moment Diagram (BMD).



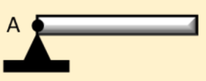
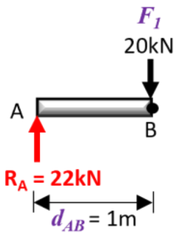
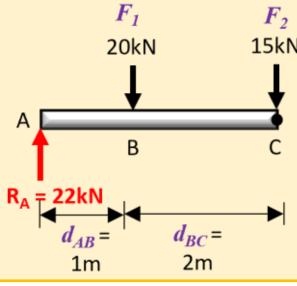
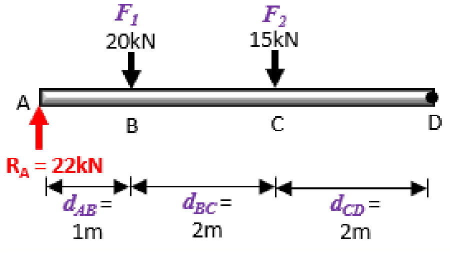

*Things to keep in mind*

"The BMD will be zero at the support of a simply supported beam".

2.0

The maximum bending moment, $M_{max}$ = 26kNm

The bending moment can also be calculated using Figure 2.3.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$ |
| A – B |   | $M_B = R_A d_{AB}$<br>$M_B = +(22kN)(1m)$<br>$M_B = 22\text{kNm}$ |
| A – C |   | $M_C = R_A d_{AC} - F_1 d_{BC}$<br>$M_C = +(22kN)(3\text{m}) - (20\text{kN})(2\text{m})$<br>$M_C = 26\text{kNm}$ |
| A – D |   | $M_C = R_A d_{AD} - F_1 d_{BD} - F_2 d_{CD}$<br>$M_D = +(22kN)(5\text{m}) - (20\text{kN})(4\text{m}) - (15\text{kN})(2\text{m})$<br>$M_D = 0$ |

## Example 2.4 Simply Supported Beam

A simply supported beam loaded as shown in Figure 2.4. If reaction at support A are $R_A$ = 22kN and $R_D$ = 13kN.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.

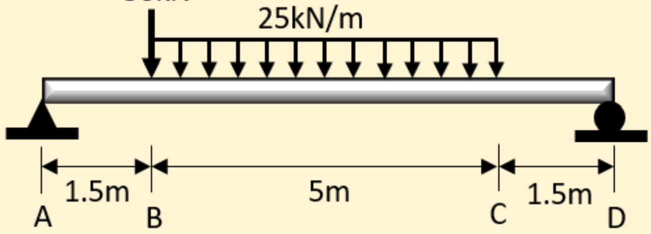
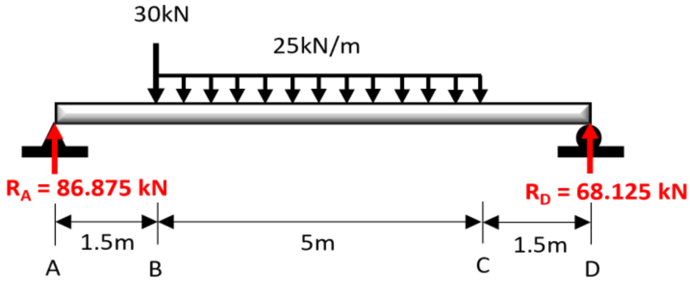
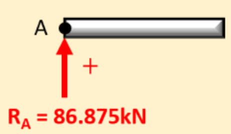
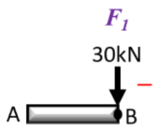
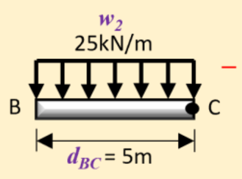
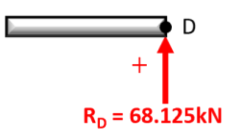



Figure 2.4

Solution:

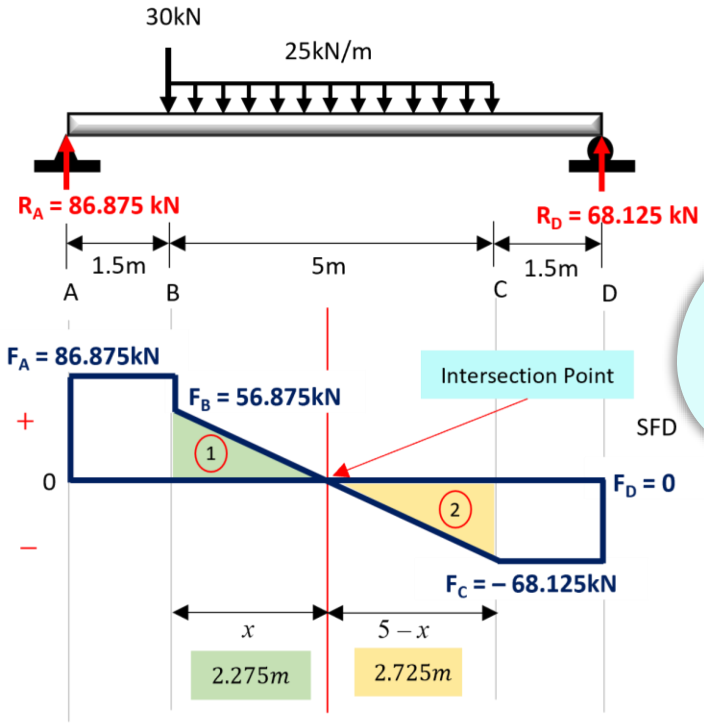



Step 1: Calculate Shear Force, kN.

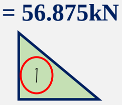
| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |  | $F_A = R_A = +86.875kN$ |
| B |  | $F_B = F_A - F_1$<br>$F_B = 86.875kN - (30\text{kN}) = 56.875kN$ |
| B – C |  | $F_C = F_B - w_2 d_{BC}$<br>$F_C = 56.875kN - \left(25\frac{kN}{m}\right)(5m)$<br>$F_C = -68.125\text{kN}$ |
| D |  | $F_D = F_C - R_D$<br>$F_D = -68.125kN + 68.125kN = 0$ |

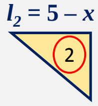
## Example 2.4 Simply Supported Beam

Step 2: Illustrate the Shear Force Diagram (SFD) .








$$\frac{l_1}{h_1} = \frac{l_2}{h_2}$$

$$\frac{x}{56.875} = \frac{5-x}{68.125}$$

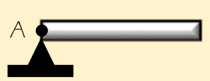
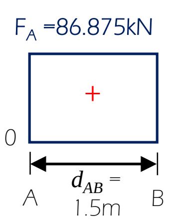
$$x = \frac{5(56.875)}{(56.875 + 68.125)}$$

$$x = 2.275m$$

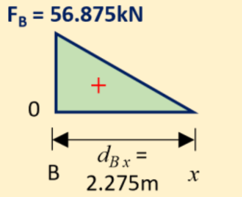
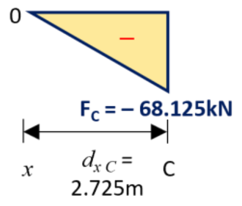
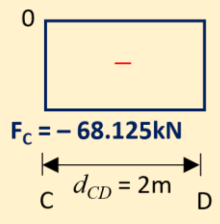
Therefore;

$$5 - x = 5 - 2.275$$
$$= 2.725m$$

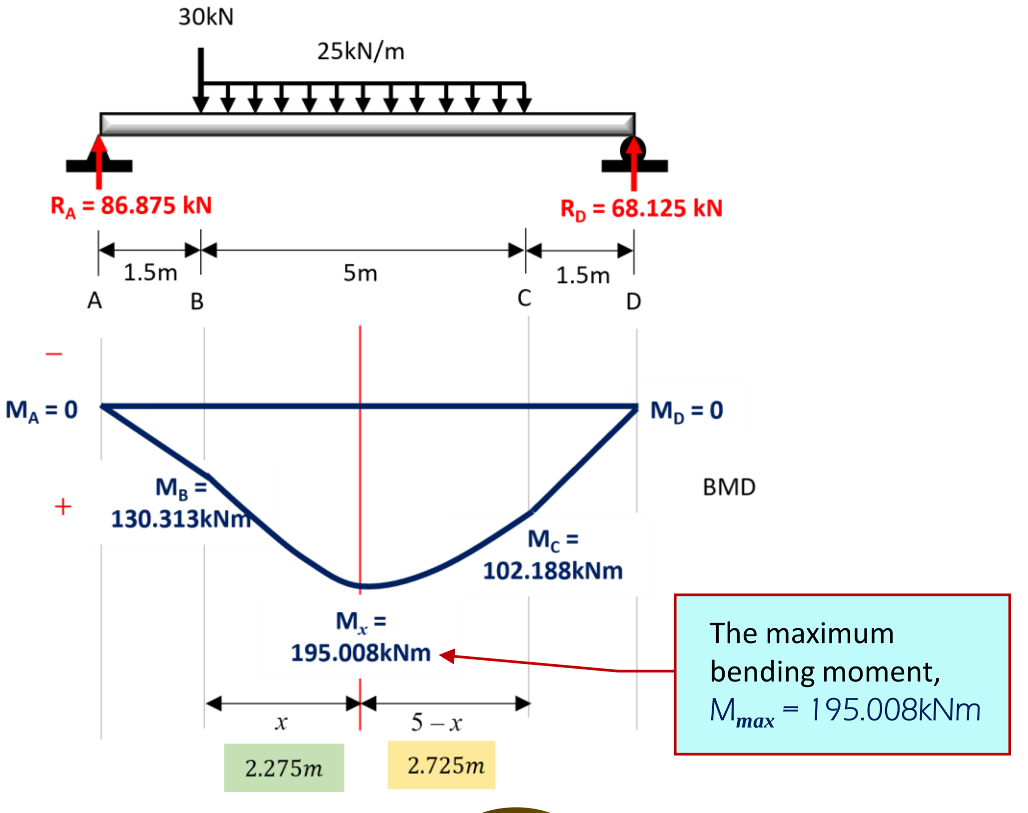
Step 3: Based on SFD in Figure 2.4, calculate Bending Moment, kNm.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$ |
| A – B |   | $M_B = F_A d_{AB}$<br>$M_B = (86.875\text{kN})(1.5m)$<br>$M_B = 130.313\text{kNm}$ |

## Example 2.4 Simply Supported Beam

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| B – $x$ |   | Area of triangle<br>$M_x = M_B + \frac{1}{2}F_B d_{B\,x}$<br>$M_x = 130.313kNm + \frac{1}{2}(56.875kN)(2.275\text{m})$<br>$M_x = 195.008\text{kNm}$ |
| $x$ – C |   | $M_C = M_x + \frac{1}{2}F_C d_{x\,C}$<br>$M_C = 195.008kNm + \frac{1}{2}(-68.125kN)(2.725\text{m})$<br>$M_C = 102.188\text{kNm}$ |
| C – D |   | $M_D = M_x + \frac{1}{2}F_C d_{x\,C}$<br>$M_D = 102.188kNm + (-68.125kN)(1.5\text{m})$<br>$M_D = 0.0005\text{kNm}$<br>$M_D \approx 0\text{kNm}$ |

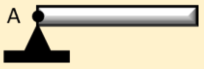
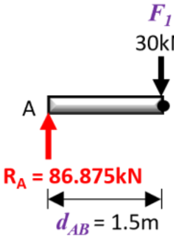
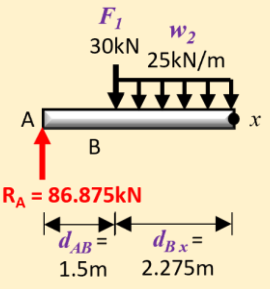
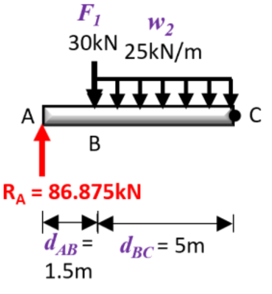
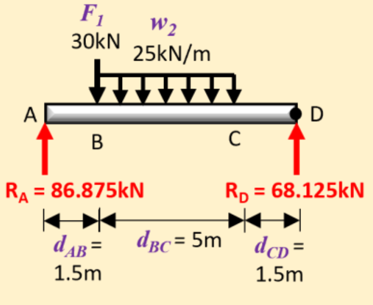
Step 4: Illustrate the Bending Moment Diagram (BMD) .



## Example 2.4 Simply Supported Beam

2.0

The bending moment can also be calculated using Figure 2.4.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |  | $M_A = 0$ |
| A – B |  | $M_B = R_A d_{AB}$<br>$M_B = +(86.875kN)(1.5m)$<br>$M_B = 130.313\text{kNm}$ |
| A – $x$ |  | $M_C = R_A d_{Ax} - F_1 d_{Bx} - w_2 d_{Bx}\left(\frac{d_{Bx}}{2}\right)$<br>$M_C = +(86.875kN)(3.775m) - (30\text{kN})(2.275\text{m})$<br>$-\left(25\frac{kN}{m}\right)(2.275m)\left(\frac{2.275m}{2}\right)$<br>$M_C = 195.093\text{kNm}$ |
| A – C |  | $M_C = R_A d_{AC} - F_1 d_{BC} - w_2 d_{BC}\left(\frac{d_{BC}}{2}\right)$<br>$M_C = +(86.875kN)(6.5m) - (30\text{kN})(5\text{m})$<br>$-\left(25\frac{kN}{m}\right)(5m)\left(\frac{5m}{2}\right)$<br>$M_C = 102.188\text{kNm}$ |
| A – D |  | $M_D = R_A d_{AD} - F_1 d_{BD} - w_2 d_{BC}\left(\frac{d_{BC}}{2} + d_{CD}\right)$<br>$M_D = +(86.875kN)(8m) - (30\text{kN})(6.5\text{m})$<br>$-\left(25\frac{kN}{m}\right)(5m)\left(\frac{5m}{2} + 1.5m\right)$<br>$M_D = 0\text{kNm}$ |

## Example 2.5 Simply Supported Beam

A simply supported beam loaded as shown in Figure 2.5.

i. Calculate reactions at the support of beam
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

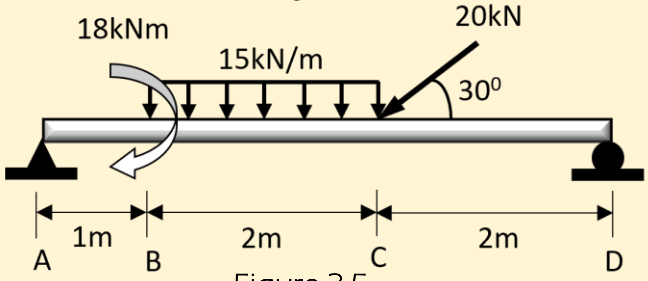
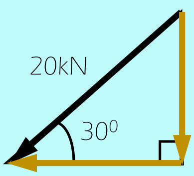



Figure 2.5

Solution:

Step 1: Sketch Free Body Diagram.

Calculate the resolution force of 20kN at an angle of 30⁰ above the horizontal using trigonometric.

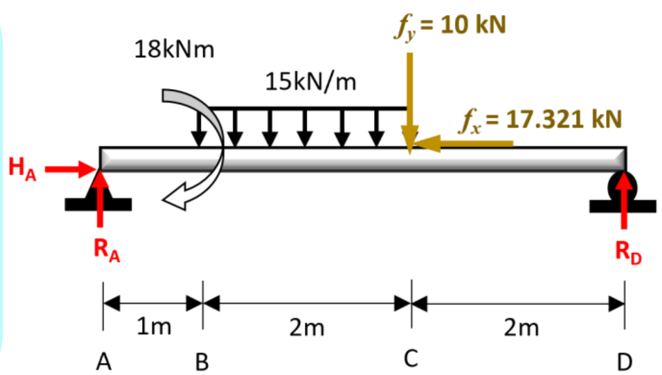



$f_y = 20Sin30^0$

$f_y = 10kN$

$f_x = 20Cos30^0$

$f_y = 17.321kN$




Step 2: Determine reaction at support A and D by using principle of equilibrium forces.

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A = 17.321kN$$

$$\Sigma M_D = 0;$$

$$18kNm + \left(15\frac{kN}{m}\right)(2m)(2m) + (10kN)(3m) - R_D(5m) = 0$$

$$R_D = 21.6kN$$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A + 21.6kN = \left(15\frac{kN}{m}\right)(2m) + 10kN$$

$$R_A = 18.4kN$$

## Example 2.5 Simply Supported Beam

Step 3: Calculate the Shear Force.

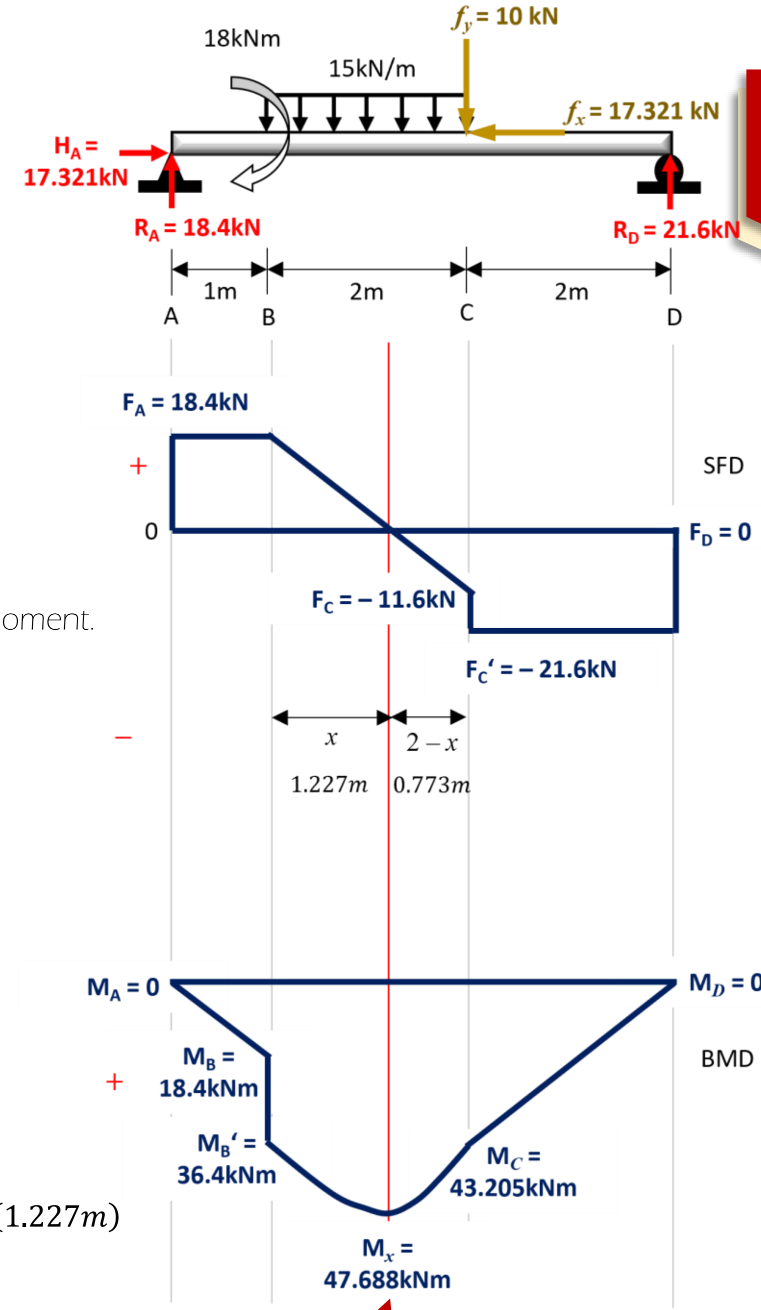
$F_A = R_A = 18.4kN$

$F_B = F_A = 18.4kN$

$$F_C = F_B - wd_{BC}$$
$$= 18.4kN - \left(15\frac{kN}{m}\right)$$
$$= -11.6\text{kN}$$

$$F_C{}' = F_C - f_y$$
$$= -11.6kN - 10\text{kN}$$
$$= -21.6\text{kN}$$

$$F_D = F_C{}' + R_D$$
$$= -21.6k\text{N} + 21.6\text{kN}$$
$$= 0$$




Step 4 : Calculate the Bending Moment.

$$\frac{x}{18.4} = \frac{2-x}{11.6}$$
$$x = 1.227m$$

$M_A = 0$

$$M_B = (18.4kN)(1\text{m})$$
$$= 18.4\text{kNm}$$

$$M_B{}' = M_B + M$$
$$= 18.4\text{kNm} + 18\text{kNm}$$
$$= 36.4\text{kNm}$$

$$M_x = M_B{}' + \tfrac{1}{2}F_B d_{Bx}$$
$$= 36.4k\text{Nm} + \tfrac{1}{2}(18.4kN)(1.227m)$$
$$= 47.688\text{kNm}$$

$$M_C = M_x + \tfrac{1}{2}F_C d_{xC}$$
$$= 47.688k\text{Nm} + \tfrac{1}{2}(-11.6kN)(0.773m)$$
$$= 43.205\text{kNm}$$

$$M_D = M_C + F_C{}' d_{CD}$$
$$= 43.205k\text{Nm} + (-21.6\text{kN})(2\text{m})$$
$$= 5 \times 10^{-3}$$
$$\approx 0$$

The maximum bending moment, $M_{max}$ = 47.688kNm

# SELF-TEST 2A

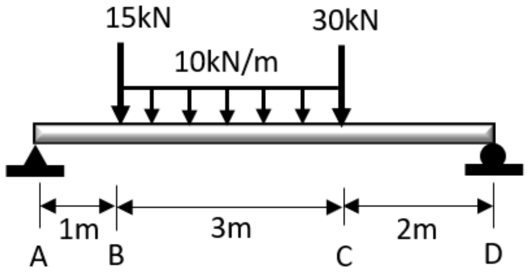
## Simply Supported Beam



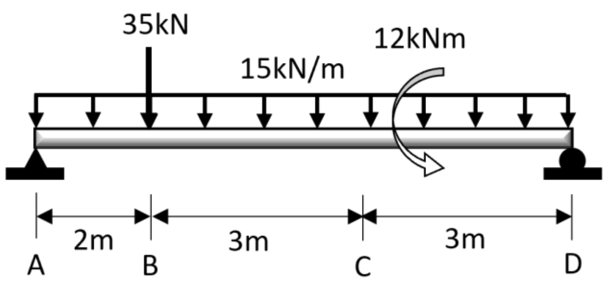
A simply supported beam loaded as shown in figure below.

i. Calculate the reactions at the support of beam.
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

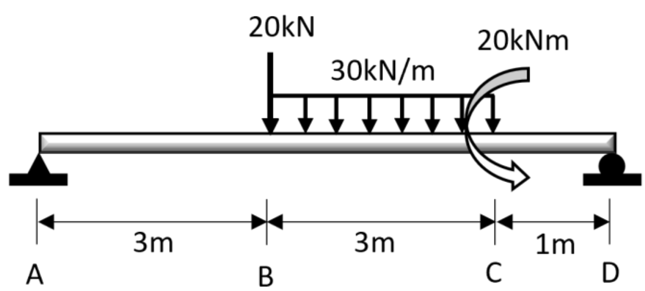
1.



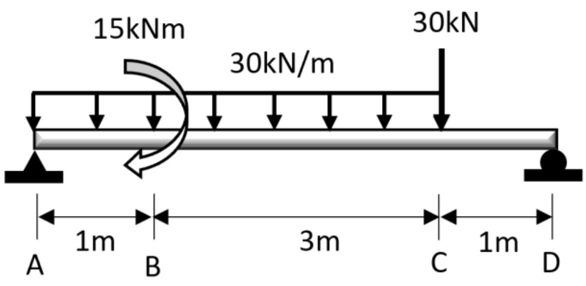

2.




3.




4.



## Example 2.6 Cantilever Beam

A cantilever beam loaded as shown in Figure 2.6. If reaction at support A are $R_A$ = 75kN and $M_A$ = 165kNm.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.

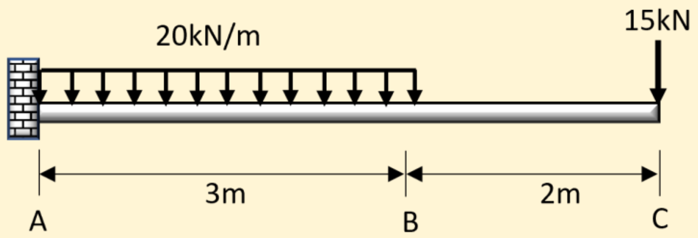
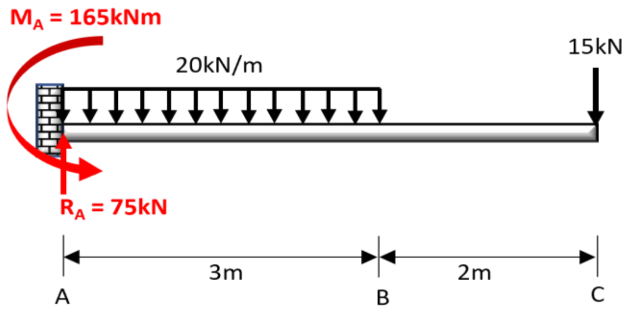
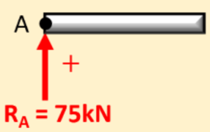
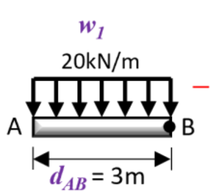
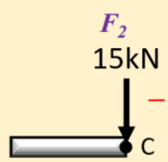



Figure 2.6

Solution:

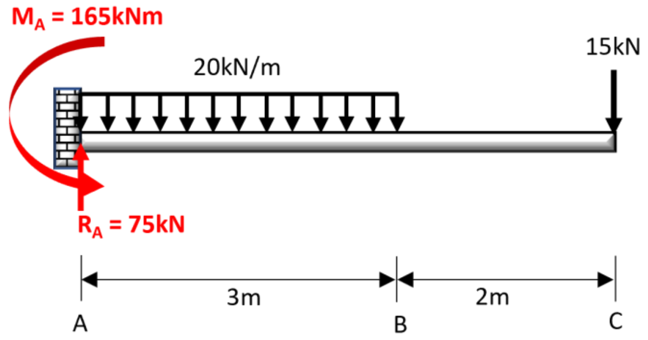


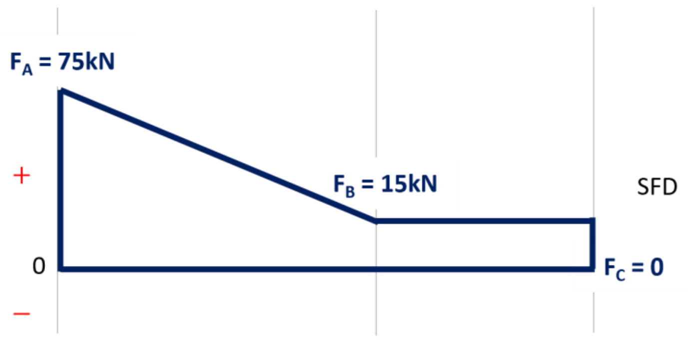

Step 1: Calculate Shear Force, kN.

| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |   | $F_A = R_A = +75kN$ |
| A – B |   | $F_B = F_A - w_1 d_{AB}$<br>$F_B = 75kN - \left(20\frac{kN}{m}\right)(3m) = 15kN$ |
| C |   | $F_C = F_B - F_2$<br>$F_C = 15kN - 15kN = 0$ |

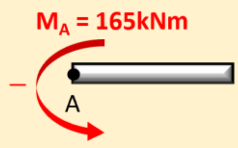
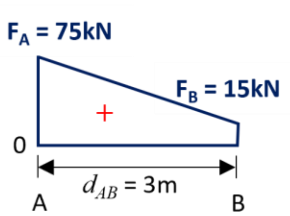
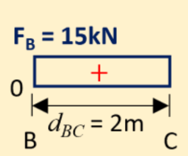
## Example 2.6 Cantilever Beam

Step 2: Illustrate the Shear Force Diagram (SFD) .



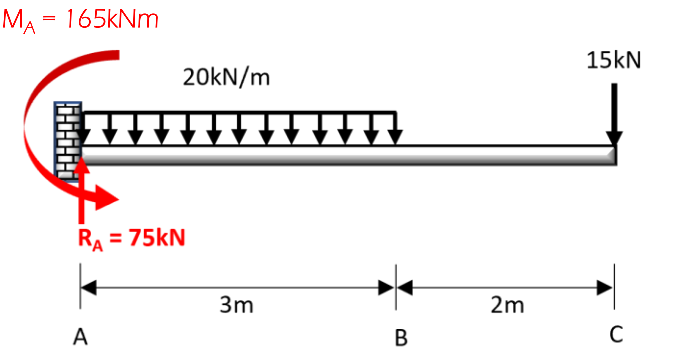



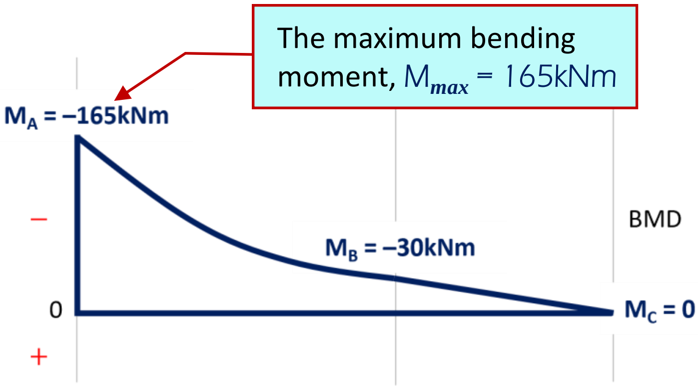

Step 3: Based on SFD in Figure 2.6, calculate Bending Moment, kNm.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |  | $M_A = -165kNm$ |
| A – B |  | $M_B = M_A + \frac{1}{2}(F_A + F_B)d_{AB}$ (Area of trapezium)<br>$M_B = -165\text{kNm} + \frac{1}{2}(75kN + 15kN)(3m)$<br>$M_B = -30\text{kNm}$ |
| B – C |  | $M_C = M_B + F_B d_{BC}$<br>$M_C = -30kN + (15kN)(2\text{m})$<br>$M_C = 0$ |

## Example 2.6 Cantilever Beam

Step 4: Illustrate the Bending Moment Diagram (BMD) .



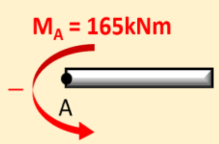
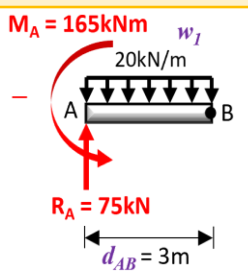
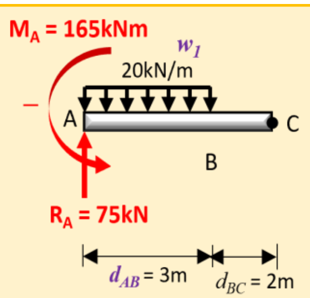






### Things to keep in mind

- **At fixed support** of a cantilever beam, the bending moment is always maximum
- **At free end** of the cantilever beam, the bending moment is always zero.

The bending moment can also be calculated using Figure 2.6.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |  | $M_A = -165kN\text{m}$ |
| A – B |  | $M_B = M_A + R_A d_{AB} - w_1 d_{AB}\dfrac{d_{AB}}{2}$<br>$M_B = -165\text{kNm} + (75kN)(3m) - 20\dfrac{kN}{m}(3m)\left(\dfrac{3m}{2}\right)$<br>$M_B = -30\text{kNm}$ |
| A – C |  | $M_C = M_A + R_A d_{AC} - w_1 d_{AB}\left(\dfrac{d_{AB}}{2} + d_{BC}\right)$<br>$M_C = -165kN + (75kN)(3\text{m} + 2\text{m})$<br>$-20\dfrac{kN}{m}(3m)\left(\dfrac{3m}{2} + 2m\right)$<br>$M_C = 0$ |

## Example 2.7 Cantilever Beam

A cantilever beam loaded as shown in Figure 2.7. If reaction at support A are $R_A$ = 75kN and $M_A$ = 165kNm.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.

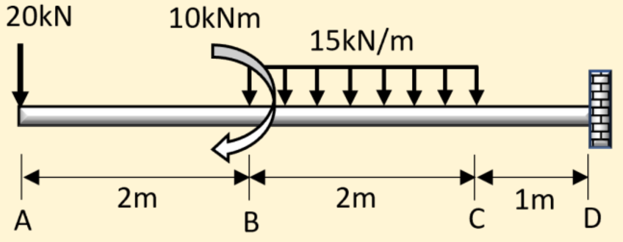
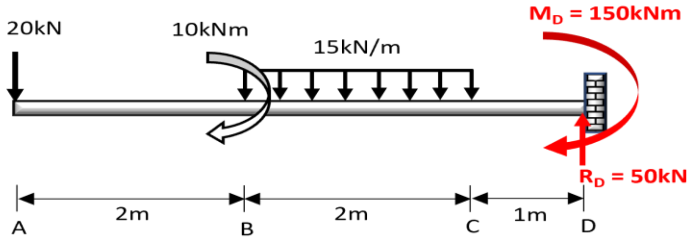


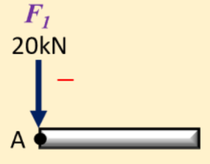
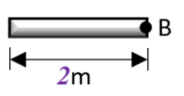
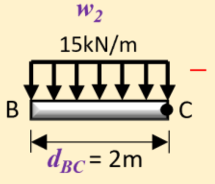
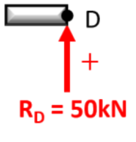

Figure 2.7



Solution:

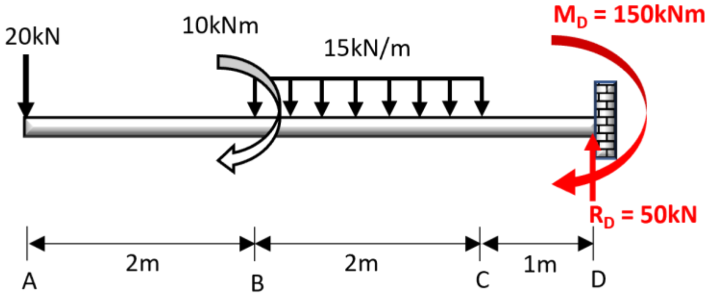


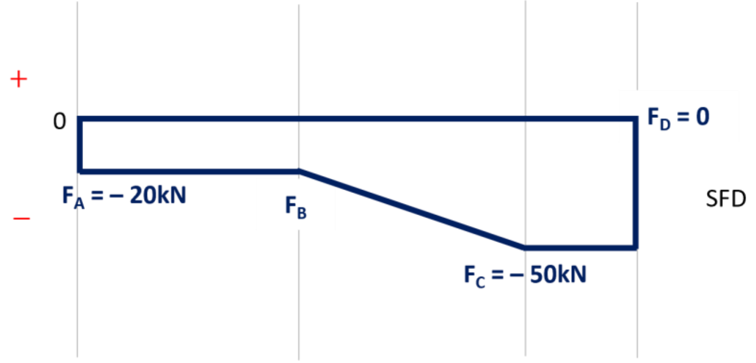

Step 1: Calculate Shear Force, kN.

| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |   | $F_A = -20kN$ |
| B |   | $F_B = F_A$<br>$F_B = -20kN$ |
| B – C |   | $F_C = F_B - w_2 d_{BC}$<br>$F_C = -20kN - \left(15\frac{kN}{m}\right)(2m) = -50\text{kN}$ |
| D |   | $F_D = F_C + F_D$<br>$F_D = -50kN + 50\text{kN} = 0$ |

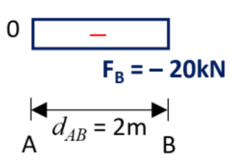
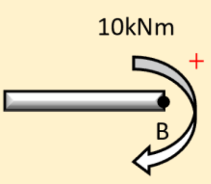
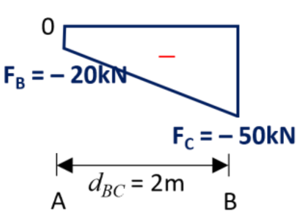
## Example 2.7 Cantilever Beam

Step 2: Illustrate the Shear Force Diagram (SFD) .

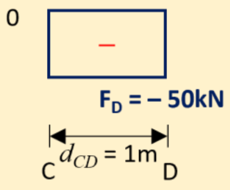
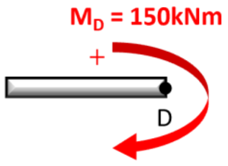


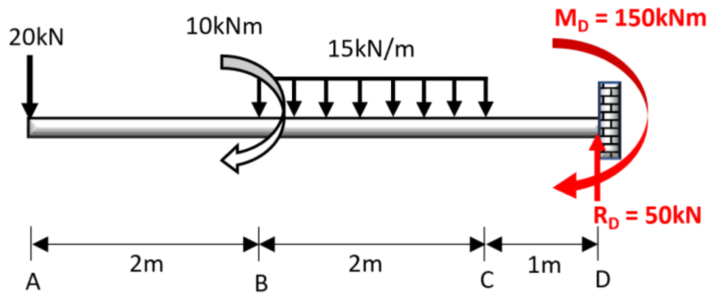

Step 3: Based on SFD in Figure 2.7, calculate Bending Moment, kNm.

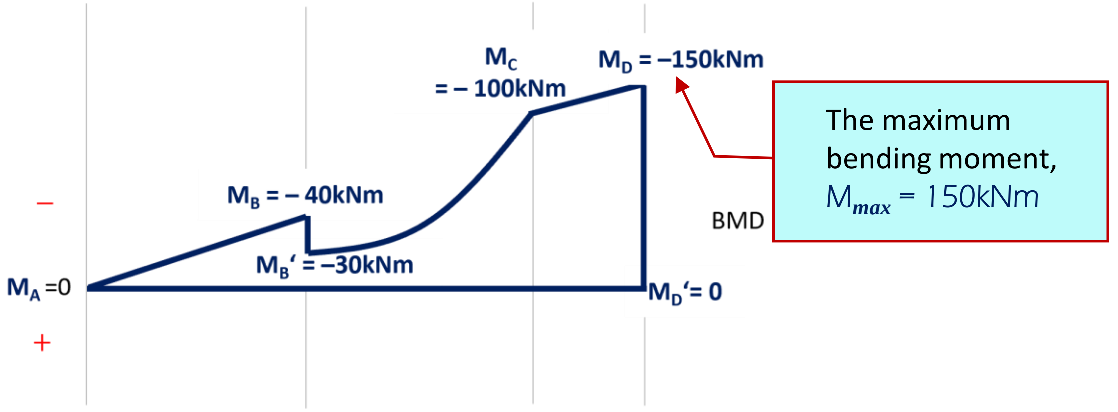
| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$ |
| A – B |   | $M_B = M_A + F_B d_{AB}$<br>$M_B = 0\text{kNm} + (-20kN)(2m)$<br>$M_B = -40\text{kNm}$ |
| B |   | $M_B{}' = M_B + \text{M}$<br>$M_B{}' = -40kN + 10\text{kNm} = 0$<br>$M_B{}' = -30\text{kNm}$ |
| B – C |   | $M_C = M_B{}' + \frac{1}{2}(F_B + F_C)d_{BC}$<br>$M_C = -30kN + \frac{1}{2}[(-20kN + (-50kN)](2m) = 0$<br>$M_C = -100\text{kNm}$ |

## Example 2.7 Cantilever Beam

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| C – D |   | $M_D = M_C + F_D d_{CD}$<br>$M_D = -100\text{kNm} + (-50kN)(1m)$<br>$M_D = -150\text{kNm}$ |
| D |   | $M_D{}' = M_D + M_D$<br>$M_D{}' = -150kN + 150\text{kNm}$<br>$M_D{}' = 0$ |

Step 4: Illustrate the Bending Moment Diagram (BMD) .

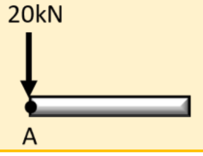
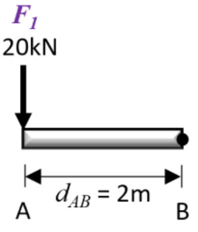
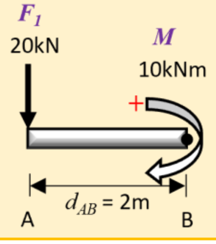
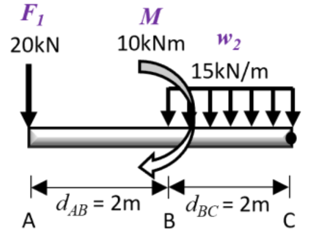
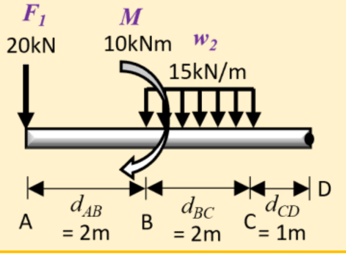
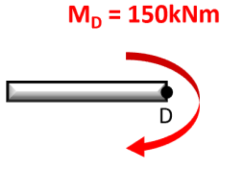


## Example 2.7 Cantilever Beam



The bending moment can also be calculated using Figure 2.7.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |  | $M_A = 0$ |
| A – B |  | $M_B = R_A d_{AB}$<br>$M_B = (-20kN)(2m)$<br>$M_B = -40\text{kNm}$ |
| B |  | $M_B' = R_A d_{AB} + M$<br>$M_B = (-20kN)(2m) + 10kNm$<br>$M_B = -30\text{kNm}$ |
| A – C |  | $M_C = F_1 d_{AC} + M - w_2 d_{BC}\left(\frac{d_{BC}}{2}\right)$<br>$M_C = (-20kN)(4\text{m}) + 10\text{kNm}$<br>$-\left(15\frac{kN}{m}\right)(2m)\left(\frac{2m}{2}\right)$<br>$M_C = -100kNm$ |
| A – D |  | $M_D = F_1 d_{AD} + M - w_2 d_{BC}\left(\frac{d_{BC}}{2} + d_{CD}\right)$<br>$M_D = (-20kN)(5m) + 10\text{kNm}$<br>$-\left(15\frac{kN}{m}\right)(2m)\left(\frac{2m}{2} + 1m\right)$<br>$M_D = -150kNm$ |
| D |  | $M_D' = M_D + M_D$<br>$M_D' = -150\text{kNm} + 150kNm$<br>$M_D{}' = 0kNm$ |

## Example 2.8 Cantilever Beam

A cantilever beam loaded as shown in Figure 2.8.

i. Calculate reactions at the support of beam
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

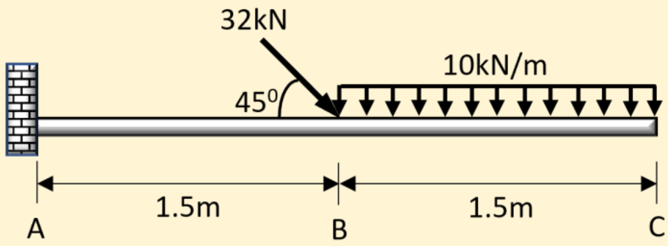



Figure 2.8

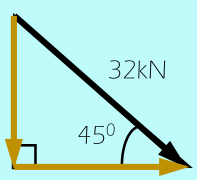
Solution:

Step 1: Sketch Free Body Diagram.

Calculate the resolution force of 32kN at an angle of $45^0$ above the horizontal using trigonometric.

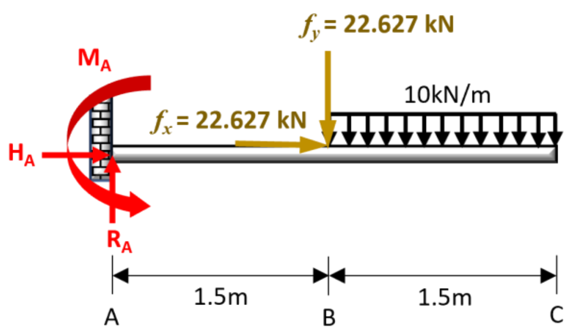
$f_y = 32Sin45^0$

$f_y = 22.627kN$




$f_x = 32Cos45^0$

$f_y = 22.627kN$




Step 2: Determine reactions at support A by using principle of equilibrium forces.

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_A + 22.627kN = 0$$

$$H_A = -22.627kN$$

Therefore, the direction of $H_A$ is to the leftward (←).

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

$$R_A = \left(10\frac{kN}{m}\right)(1.5m) + 22.627kN$$

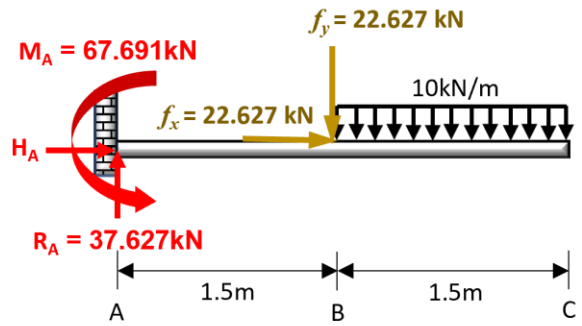
$$R_A = 37.627kN$$

$$\Sigma M_D = 0;$$

$$(22.627kN)(1.5m) + \left(10\frac{kN}{m}\right)(1.5m)\left(\frac{1.5m}{2} + 1.5m\right) - M_A = 0$$

$$M_A = 67.691kNm$$

## Example 2.8 Cantilever Beam

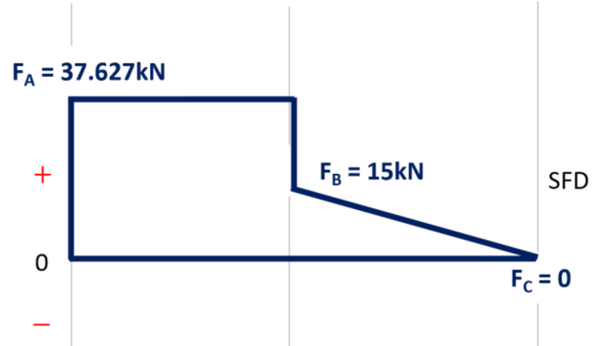



Step 3: Calculate the Shear Force.

$$F_A = R_A = 37.627kN$$

$$\begin{aligned} F_B &= F_A - f_y \\ &= 37.627kN - 22.627\text{kN} \\ &= 15\text{kN} \end{aligned}$$

$$\begin{aligned} F_C &= F_B - wd_{BC} \\ &= 15kN - \left(10\frac{kN}{m}\right)(1.5m) \\ &= 0 \end{aligned}$$

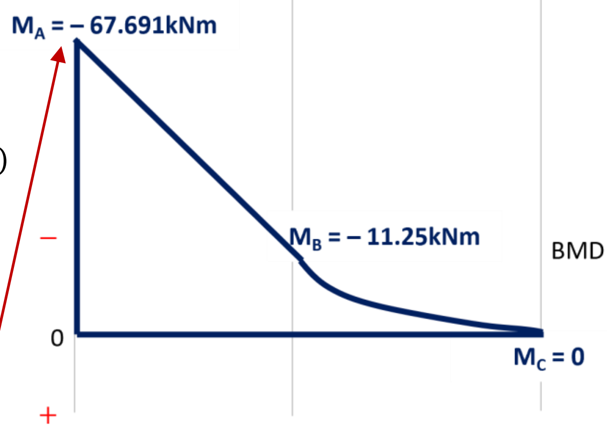



Step 4 : Calculate the Bending Moment.

$$M_A = -67.691\text{kNm}$$

$$\begin{aligned} M_B &= M_A + F_B d_{AB} \\ &= -67.691\text{kNm} + (37.627kN)(1.5\text{m}) \\ &= -11.25\text{kNm} \end{aligned}$$

$$\begin{aligned} M_C &= M_B + \tfrac{1}{2} F_C d_{BC} \\ &= -11.25kNm - \tfrac{1}{2}(-15kN)(1.5m) \\ &= 0 \end{aligned}$$




The maximum bending moment, $M_{max}$ = 67.691kNm

# SELF-TEST 2B

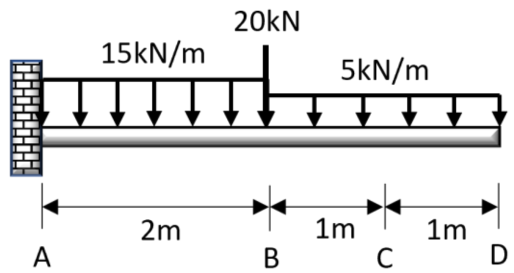
## Cantilever Beam



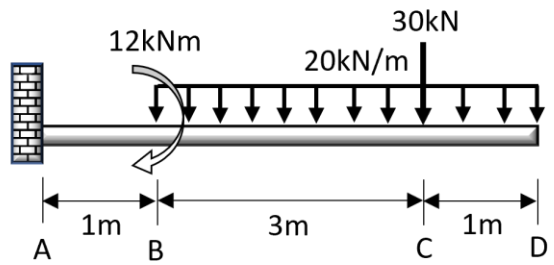
A cantilever beam loaded as shown in figure below.

i. Calculate the reaction at the support of beam.
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

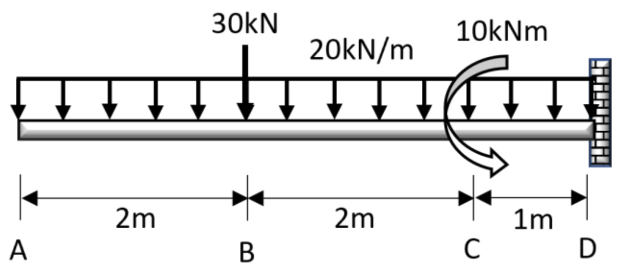
1.



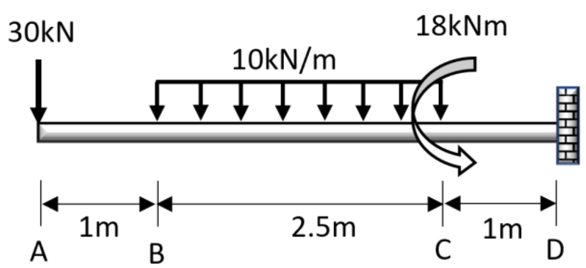

2.




3.




4.



## Example 2.9 Overhanging Beam

An overhanging loaded as shown in Figure 2.9. If reaction at support A are $R_A$ = 32.5kN and $R_D$ = 62.5kN.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.

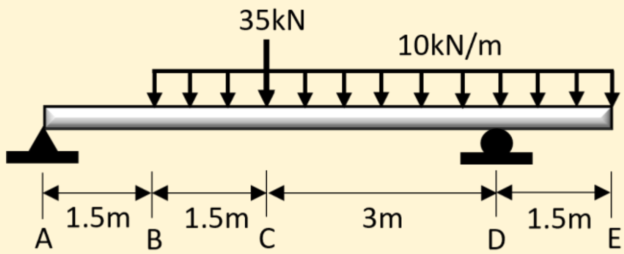
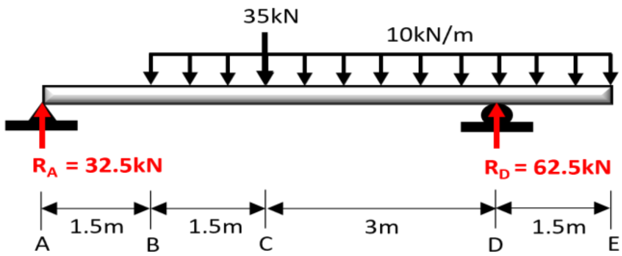


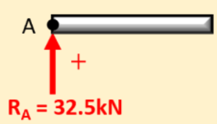
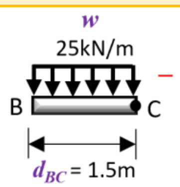
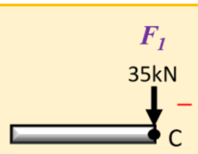
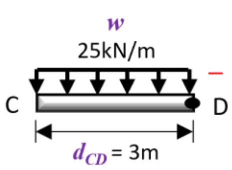

Figure 2.9



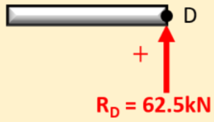
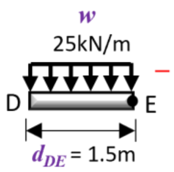
Solution:



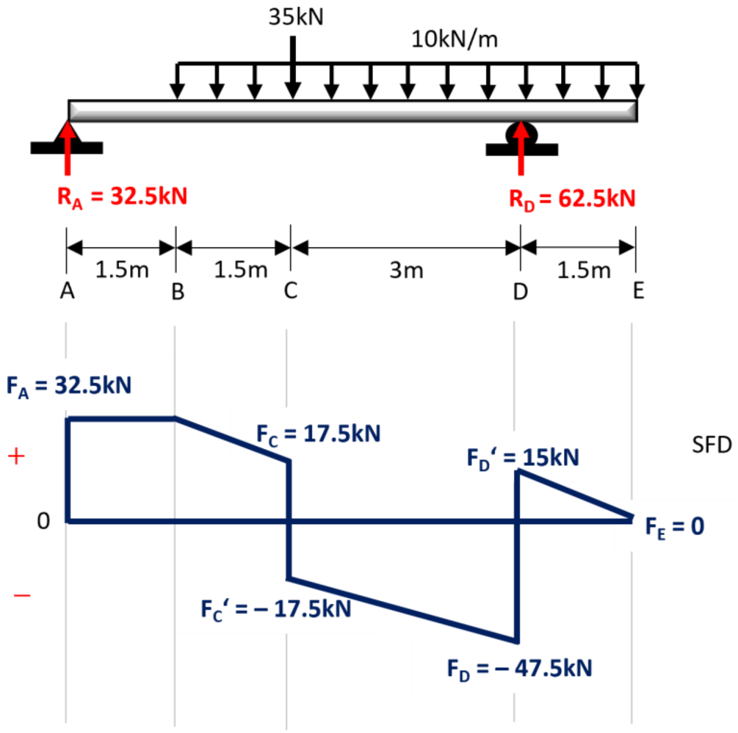

Step 1: Calculate Shear Force, kN.

| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |   | $F_A = R_A = +32.5kN$ |
| C |   | $F_C = F_B - d_{BC}$ where, $F_B = F_A = 32.5\text{kN}$<br>$F_C = 32.5kN - \left(10\frac{kN}{m}\right)(1.5m) = 17.5kN$ |
| C′ |   | $F_C' = F_C - F_1$<br>$F_C' = 17.5kN - 35kN = -17.5\text{kN}$ |
| D |   | $F_D = F_C' - wd_{CD}$<br>$F_D = -17.5kN - \left(10\frac{kN}{m}\right)(3m) = -47.5\text{kN}$ |

## Example 2.9 Overhanging Beam

| Point/ Interval | Figure | Shear Force |
|---|---|---|
| D′ |   | $F_D' = F_D + R_D$<br>$F_D' = -47.5kN + (62.5kN) = 15\text{kN}$ |
| E |   | $F_E = F_D' - wd_{BC}$<br>$F_E = 15kN - \left(10\frac{kN}{m}\right)(1.5m) = 0$ |

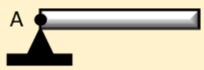
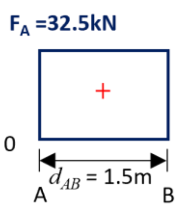
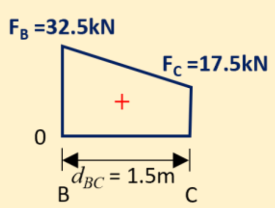
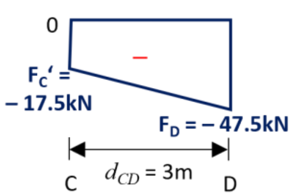
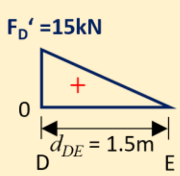
Step 2: Illustrate the Shear Force Diagram (SFD) .



## Example 2.9 Overhanging Beam

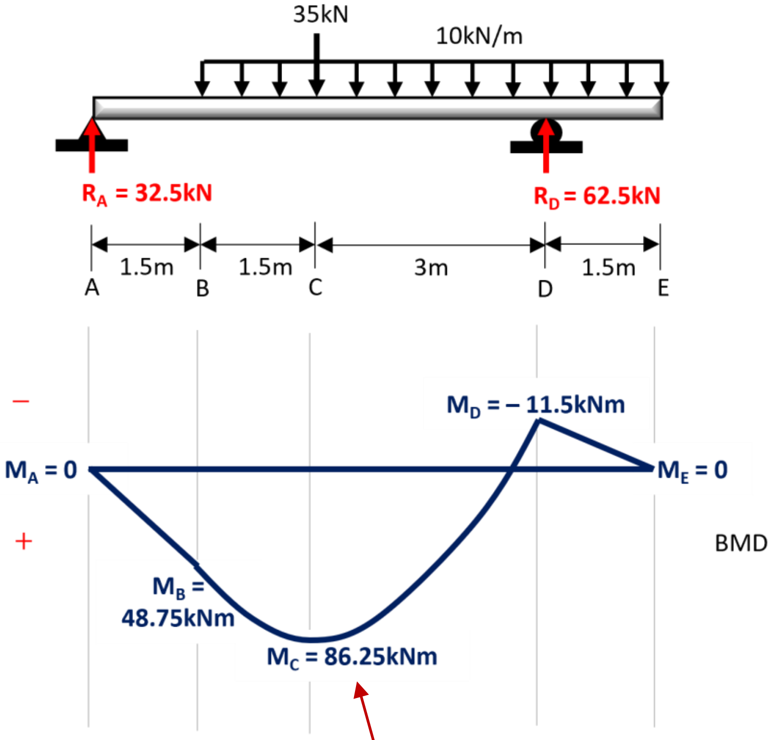
2.0

Step 3: Based on SFD in Figure 2.9, calculate Bending Moment, kNm.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$ |
| A – B |   | $M_B = F_A d_{AB}$<br>$M_B = (32.5\text{kN})(1.5m)$<br>$M_B = 48.75\text{kNm}$ |
| B – C |   | $M_C = M_B + \frac{1}{2}(F_B + F_C)d_{BC}$<br>$M_C = 48.75kN\text{m} + \frac{1}{2}(32.5kN + 17.5kN)(1.5\text{m})$<br>$M_C = 86.25\text{kNm}$ |
| C – D |   | $M_D = M_C + \frac{1}{2}(F_C' + F_D)d_{CD}$<br>$M_D = 68.25kN\text{m} + \frac{1}{2}[(-17.5kN + (-47.5kN)](3\text{m})$<br>$M_D = -11.25\text{kNm}$ |
| D – E |   | $M_E = M_D + \frac{1}{2}(F_D')d_{DE}$<br>$M_E = -11.25kN\text{m} + \frac{1}{2}(15kN)(1.5\text{m})$<br>$M_E = 0\text{kNm}$ |

## Example 2.9 Overhanging Beam

Step 4: Illustrate the Bending Moment Diagram (BMD) .

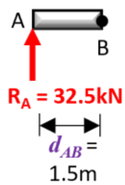
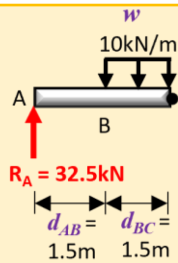
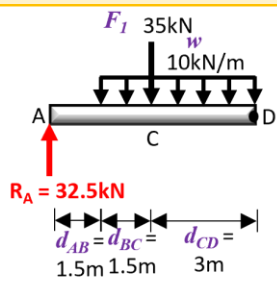
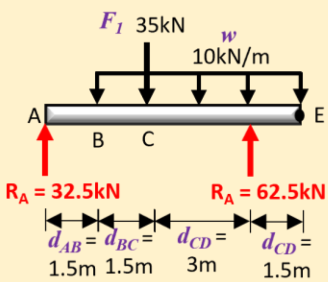



The maximum bending moment, $M_{max}$ = 86.25kNm

# Example 2.9 Overhanging Beam



The bending moment can also be calculated using Figure 2.9.

| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |  | $M_A = 0$ |
| A – B |  | $M_B = R_A d_{AB}$<br>$M_B = +(32.5kN)(1.5m)$<br>$M_B = 48.75\text{kNm}$ |
| A – C |  | $M_C = R_A d_{AC} - w_2 d_{BC}\left(\frac{d_{BC}}{2}\right)$<br>$M_C = +(32.5kN)(3m) - \left(10\frac{kN}{m}\right)(1.5m)\left(\frac{1.5m}{2}\right)$<br>$M_C = 86.25\text{kNm}$ |
| A – D |  | $M_D = R_A d_{AD} - F_1 d_{CD} - w d_{BD}\left(\frac{d_{BD}}{2}\right)$<br>$M_D = +(32.5kN)(6m) - (35kN)(3m)$<br>$-\left(10\frac{kN}{m}\right)(4.5m)\left(\frac{4.5m}{2}\right)$<br>$M_D = -11.25\text{kNm}$ |
| A – E |  | $M_E = R_A d_{AE} - F_1 d_{CE} - w d_{BE}\left(\frac{d_{BE}}{2}\right) + R_D d_{DE}$<br>$M_E = +(32.5kN)(7.5m) - (35kN)(4.5m)$<br>$-\left(10\frac{kN}{m}\right)(6m)\left(\frac{6m}{2}\right) + (62.5kN)(1.5m)$<br>$M_E = 0$ |

## Example 2.10 Overhanging Beam

An overhanging loaded as shown in Figure 2.10. If reaction at support A are $R_A$ = 54.6kN and $R_E$ = 25.4kN.

i. Calculate the value of shear force and bending moment.
ii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iii. Determine the maximum bending moment.

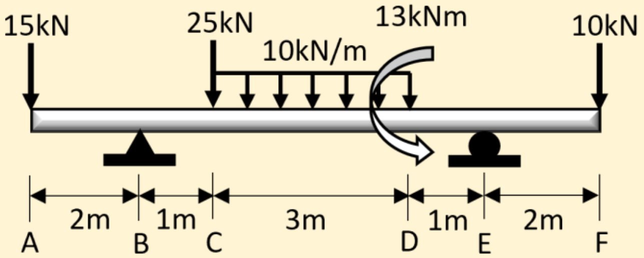
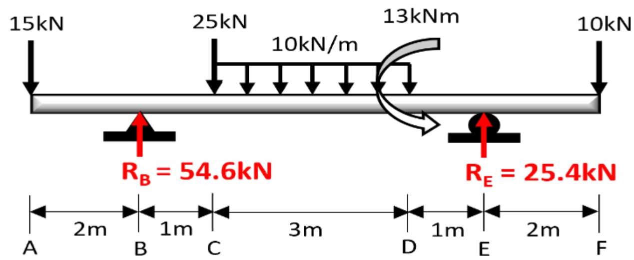
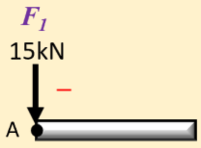
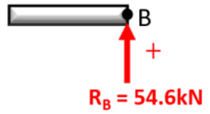
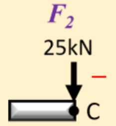
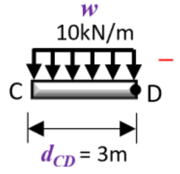



Figure 2.10

Solution:

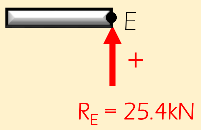
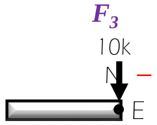



Step 1: Calculate Shear Force, kN.

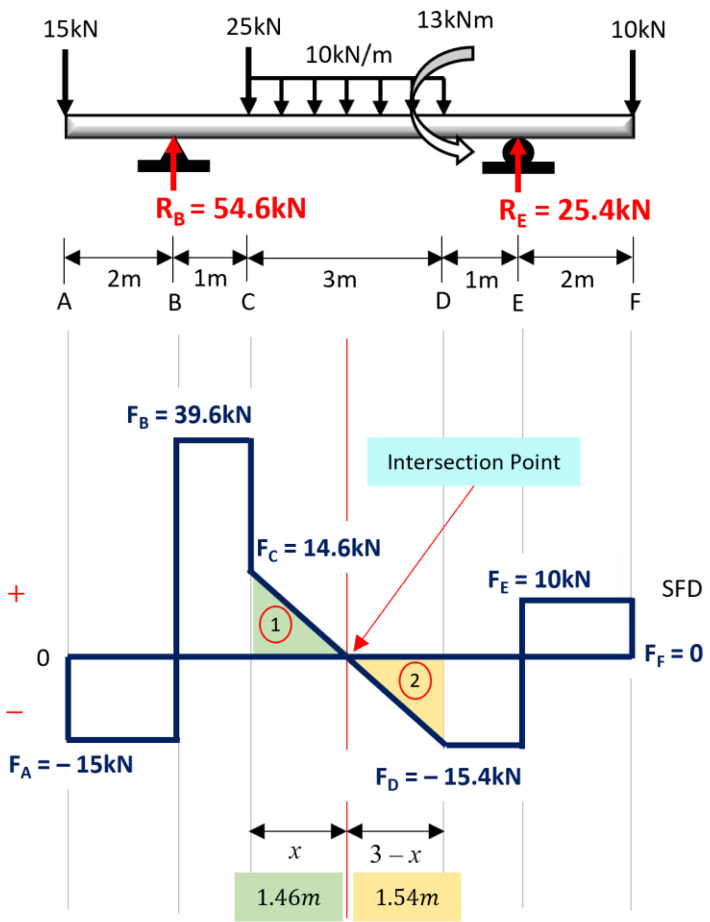
| Point/ Interval | Figure | Shear Force |
|---|---|---|
| A |   | $F_A = -F_1 = -15kN$ |
| B |   | $F_B = F_A + R_B$<br>$F_B = -15kN + (54.6\text{kN}) = 39.6kN$ |
| C |   | $F_C = F_B - F_2$<br>$F_C = 39.6kN - 25kN = 14.6\text{kN}$ |
| C – D |   | $F_D = F_C - wd_{CD}$<br>$F_D = 14.6kN - \left(10\frac{kN}{m}\right)(3\text{m}) = -15.4\text{kN}$ |

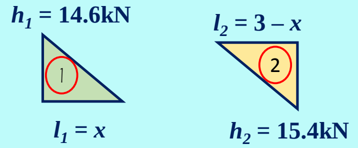
# Example 2.10 Overhanging Beam



| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| E |   | $F_E = F_D + R_E$<br>$F_E = -15.4kN + (25.4\text{kN}) = 10\text{kN}$ |
| F |   | $F_F = F_E - F_3$<br>$F_F = 10kN - (10\text{kN}) = 0$ |

Step 2: Illustrate the Shear Force Diagram (SFD) .




$$\frac{l_1}{h_1} = \frac{l_2}{h_2}$$

$$\frac{x}{14.6} = \frac{3-x}{15.4}$$

$$x = \frac{3(14.6)}{(14.6 + 15.4)}$$

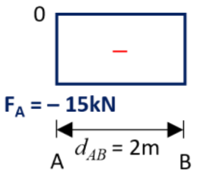
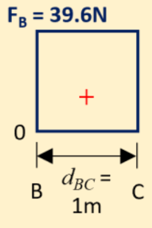
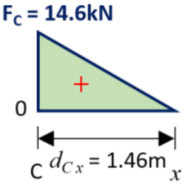
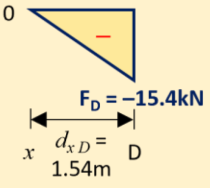
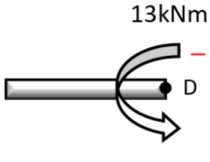
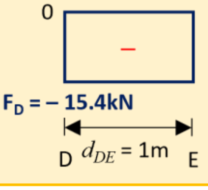
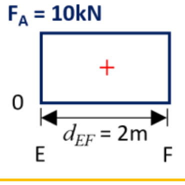
$$x = 1.46m$$

Therefore;

$$3 - x = 3 - 1.46 = 1.54m$$

# Example 2.10 Overhanging Beam

Step 3: Based on SFD in Figure 2.10, calculate Bending Moment, kNm.

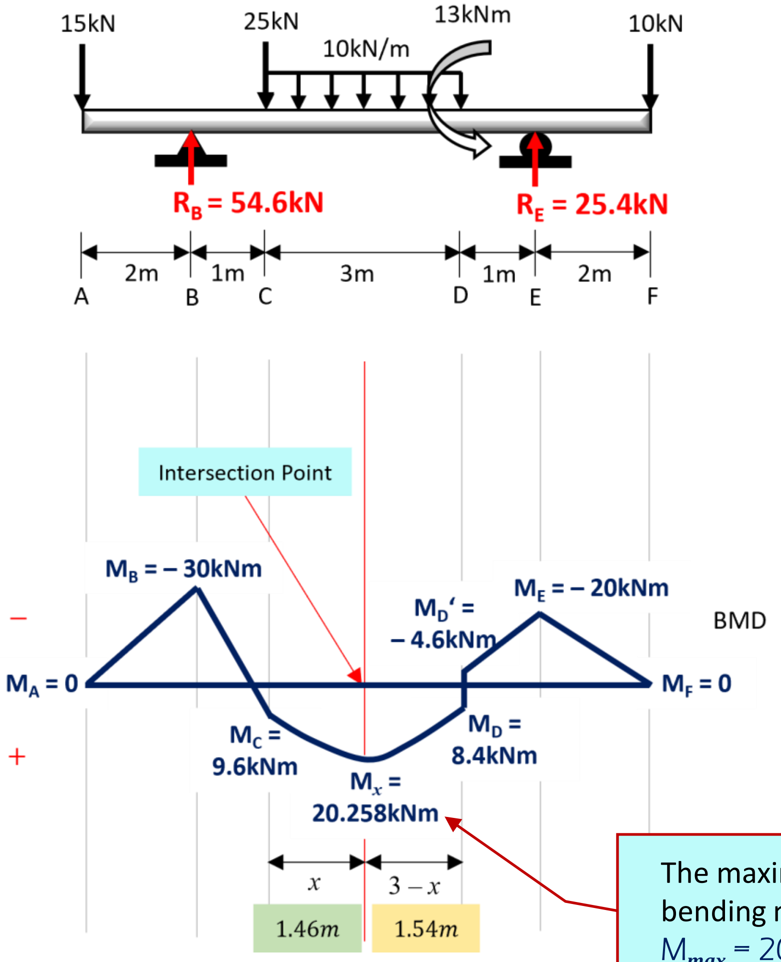
| Point/ Interval | Figure | Bending Moment |
|---|---|---|
| A |   | $M_A = 0$ |
| A – B |   | $M_B = F_A d_{AB}$<br>$M_B = (-15\text{kN})(2m)$<br>$M_B = -30\text{kNm}$ |
| B – C |   | $M_C = M_B + F_B d_{BC}$<br>$M_C = -30\text{kN}m + (39.6kN)(1m)$<br>$M_C = 9.6\text{kNm}$ |
| C – $x$ |   | $M_x = M_C + \frac{1}{2} F_C d_{Cx}$<br>$M_x = 9.6\text{kN}m + \frac{1}{2}(14.6kN)(1.46m)$<br>$M_C = 20.258\text{kNm}$ |
| $x$ – D |   | $M_D = M_C + \frac{1}{2} F_D d_{xD}$<br>$M_D = 20.258\text{kN}m + \frac{1}{2}(-15.4kN)(1.54m)$<br>$M_D = 8.4\text{kNm}$ |
| D |   | $M_D{}' = M_D - \text{M}$<br>$M_D{}' = 8.4\text{kN}m - 13\text{kNm}$<br>$M_D{}' = -4.6\text{kNm}$ |
| D – E |   | $M_E = M_D{}' + F_A d_{DE}$<br>$M_E = -4.6\text{kNm} + (-15.4\text{kN})(1m)$<br>$M_E = -20\text{kNm}$ |
| E – F |   | $M_F = M_E + F_E d_{EF}$<br>$M_F = -20\text{kNm} + (10\text{kN})(1m)$<br>$M_F = 0$ |

2.0

# Example 2.10 Overhanging Beam



Step 4: Illustrate the Bending Moment Diagram (SFD) .




The maximum bending moment, $M_{max}$ = 20.258kNm

## Example 2.10 Overhanging Beam

The bending moment can also be calculated using Figure 2.10.

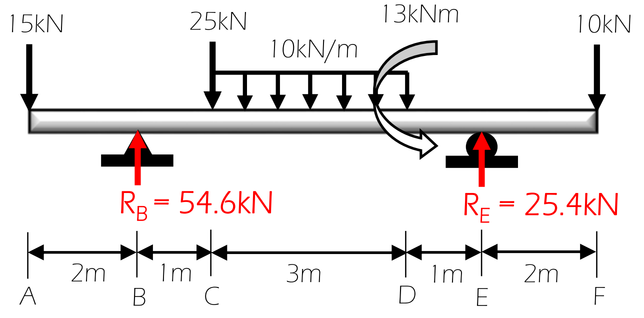



Figure 2.10

| Point/ Interval | Bending Moment |
|---|---|
| A | $M_A = 0$ |
| A – B | $M_B = F_1 d_{AB} = -(15kN)(2m) = -30\text{kNm}$ |
| A – C | $M_C = F_1 d_{AC} - R_B d_{BC} = -(15kN)(3m) + (54.6kN)(1m) = 9.6kNm$ |
| A – D | $M_D = F_1 d_{AD} + R_B d_{BD} - F_2 d_{CD} - w(d_{CD})\left(\frac{d_{CD}}{2}\right)$<br>$M_D = -(15kN)(6m) + (54.6kN)(4m) - (25kN)(3m) - \left(10\frac{kN}{m}\right)(3m)\left(\frac{3m}{2}\right)$<br>$M_D = 8.4\text{kNm}$ |
| D | $M_D' = M_D + M = 8.4kNm - 13kNm = -4.6\text{kNm}$ |
| A – E | $M_E = F_1(d_{AE}) + R_B(d_{BE}) - F_2(d_{CE}) - w(d_{CD})\left(\frac{d_{CD}}{2} + d_{DE}\right) - M$<br>$M_E = -(15kN)(7m) + (54.6kN)(5m) - (25kN)(4m) - \left(10\frac{kN}{m}\right)(3m)\left(\frac{3m}{2} + 1m\right)$<br>$-13\text{kNm}$<br>$M_E = -20\text{kNm}$ |
| A – F | $M_F = F_1 d_{AF} + R_B d_{BF} - F_2 d_{CF} - w d_{CD}\left(\frac{d_{CD}}{2} + d_{DF}\right) - M + R_E d_{EF}$<br>$M_F = -(15kN)(9m) + (54.6kN)(7m) - (25kN)(6m) - \left(10\frac{kN}{m}\right)(3m)\left(\frac{3m}{2} + 3m\right)$<br>$-13\text{kNm} + (25.4\text{kN})(2\text{m})$<br>$M_F = 0\text{kNm}$ |

## Example 2.11 Overhanging Beam

An overhanging loaded as shown in Figure 2.11.

i. Calculate reactions at the support of beam
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

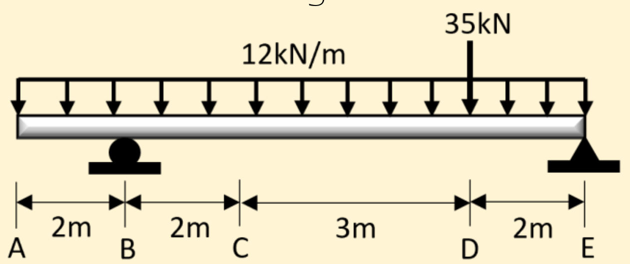
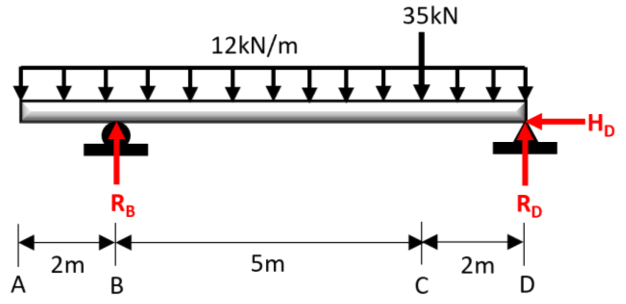



Figure 2.11



Solution:

Step 1: Sketch Free Body Diagram.




Step 2: Determine reaction at support B and D by using principle of equilibrium forces.

$$\Sigma f_x \rightarrow = \Sigma f_x \leftarrow;$$

$$H_D = 0$$

$$\Sigma M_D = 0;$$

$$-\left(12\frac{kN}{m}\right)(2m)\left(\frac{2}{2}m\right) + \left(12\frac{kN}{m}\right)(7m)\left(\frac{7}{2}m\right) + (35kN)(5m) - R_D(7m) = 0$$

$$R_D = 63.571kN$$

$$\Sigma f_y \uparrow = \Sigma f_y \downarrow;$$

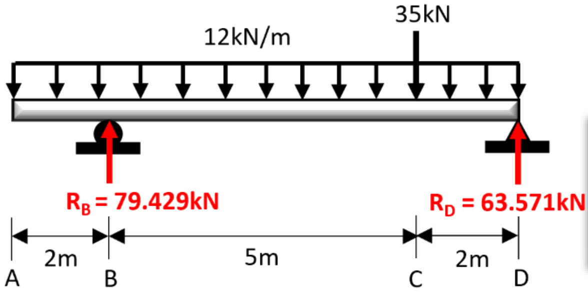
$$R_B + 63.571kN = \left(12\frac{kN}{m}\right)(9m) + 35kN$$

$$R_B = 79.429kN$$

## Example 2.11 Overhanging Beam



Step 3: Calculate the Shear Force.




$F_A = 0$

$F_B = wd_{AB}$
$= -\left(12\frac{kN}{m}\right)(2m)$
$= -24kN$

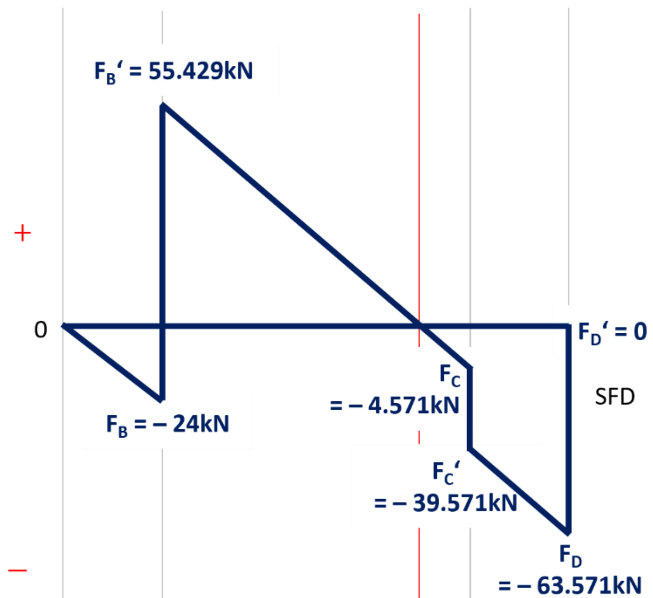
$F_B{}' = F_B + R_B$
$= -24 + 79.429kN$
$= 55.429\text{kN}$

$F_C = F_B{}' - wd_{BC}$
$= 55.429kN - \left(12\frac{kN}{m}\right)(5m)$
$= -4.571\text{kN}$

$F_C{}' = F_C - F$
$= -4.571\text{N} - 35\text{kN}$
$= -39.571\text{kN}$

$F_D = F_C{}' - wd_{CD}$
$= -39.571kN - \left(12\frac{kN}{m}\right)(2m)$
$= -63.571\text{kN}$

$F_D{}' = F_D + R_E$
$= -63.571kN - 63.571kN$
$= 0\text{kN}$

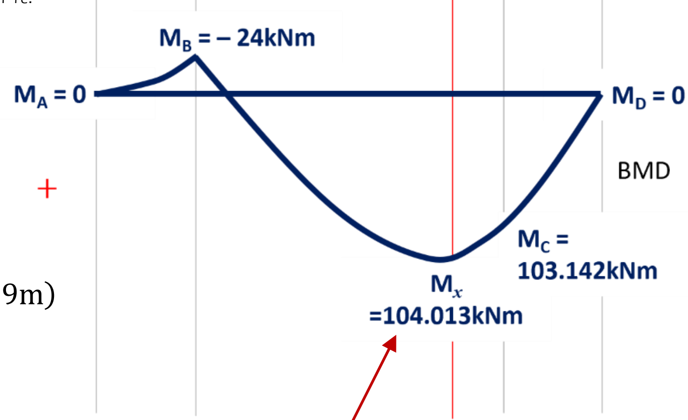



Step 4 : Calculate the Bending Moment.

$M_A = 0$

$M_B = \frac{1}{2}(-24kN)(2\text{m})$
$= -24\text{kNm}$

$M_x = M_B + \frac{1}{2}(F_B{}')d_{Bx}$
$= -24\text{kNm} + \frac{1}{2}(55.429kN)(4.619\text{m})$
$= 104.013\text{kNm}$




$M_C = M_x + \frac{1}{2}(F_C)d_{xC}$
$= 104.013\text{kN}m + \frac{1}{2}(-4.571kN)(0.381\text{m})$
$= 103.142\text{kNm}$

The maximum bending moment, $M_{max}$ = 104.013kNm

$M_D = M_C + \frac{1}{2}(F_C{}' + F_D)d_{CD}$
$= 103.142k\text{Nm} + \frac{1}{2}(-39.571kN - 63.571)\ (2\text{m})$
$= 0$

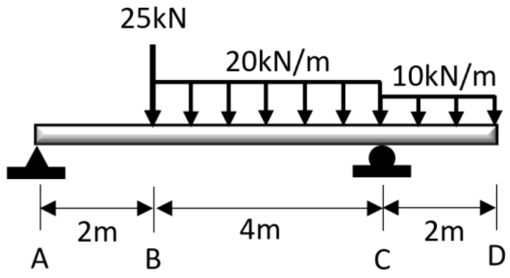
# SELF-TEST 2C Overhanging Beam



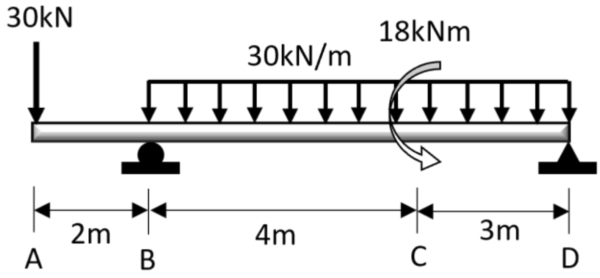
An overhanging beam loaded as shown in figure below.

i. Calculate the reactions at the support of beam.
ii. Calculate the value of shear force and bending moment.
iii. Illustrate the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) that indicate all the values at important points.
iv. Determine the maximum bending moment.

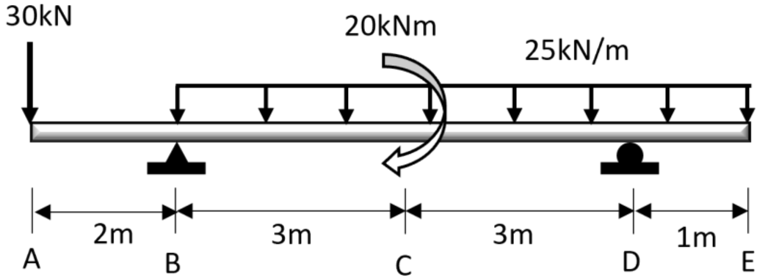
1.



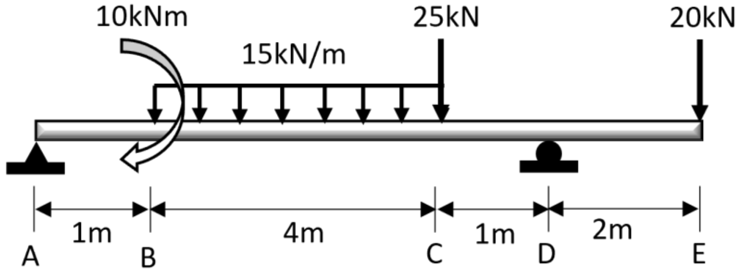

2.




3.




4.



2.0

# SUMMARY

It is important to know how the shear forces and bending moments vary along the length of a beam that is being designed.

The Employing these diagrams, the maximum and minimum shear force and bending moment are easily identified and located.

Test your knowledge and understanding here

Digital QUIZ

## A SIMPLY SUPPORTED BEAM

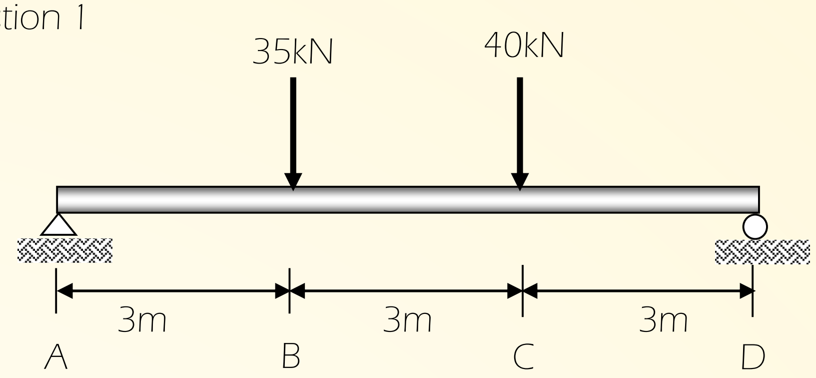
Consider a simply supported beam as shown in figure below.

i. Calculate the reactions at support of the beam.

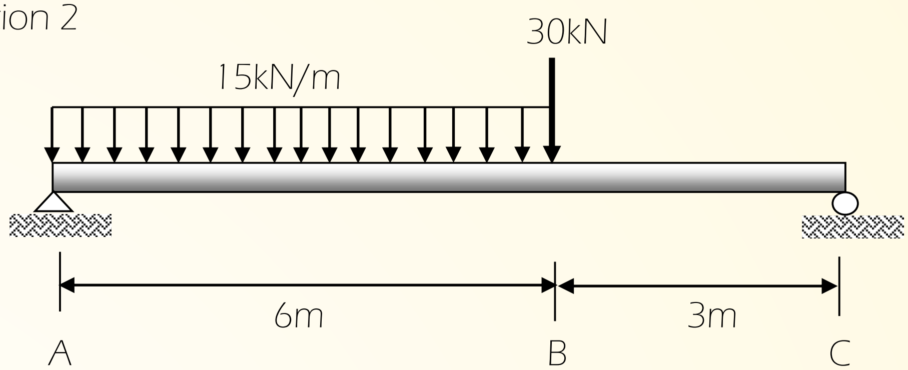
ii. Draw the shear force diagram (SFD) and bending moment diagram (BMD).

iii. Hence, determine the value of maximum bending moment

Question 1




Question 2



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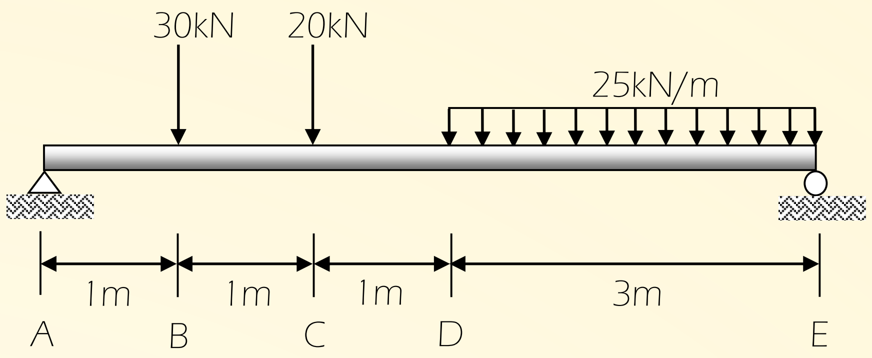


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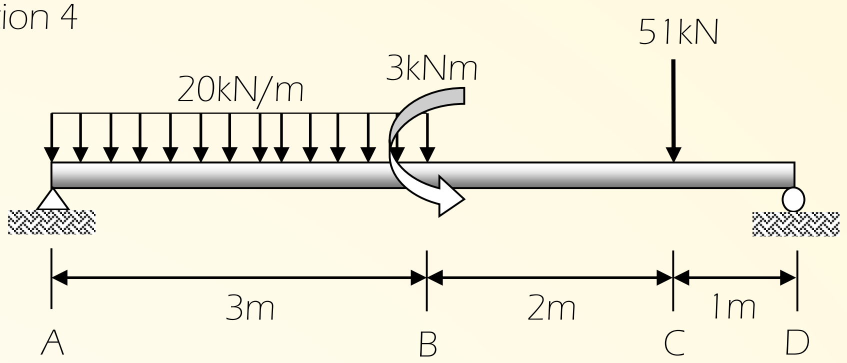


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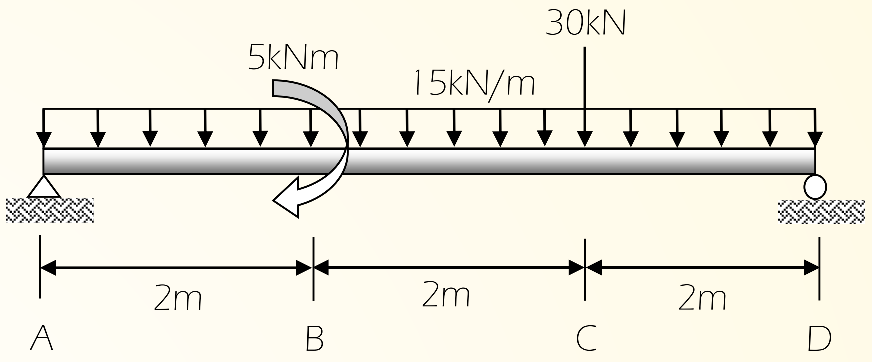

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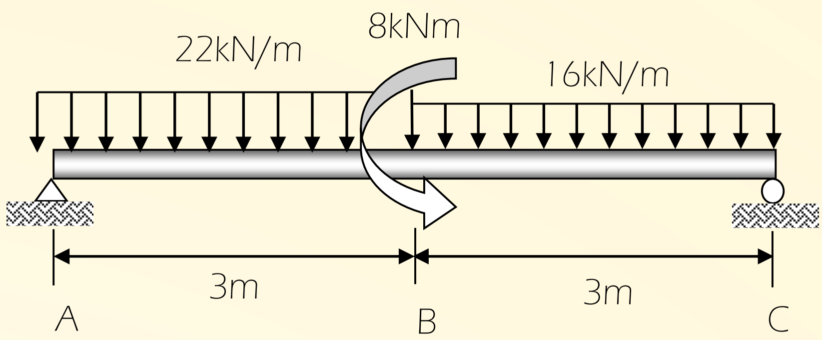


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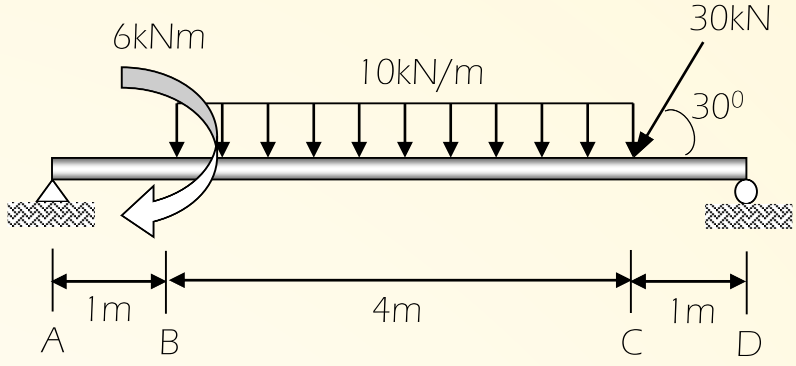


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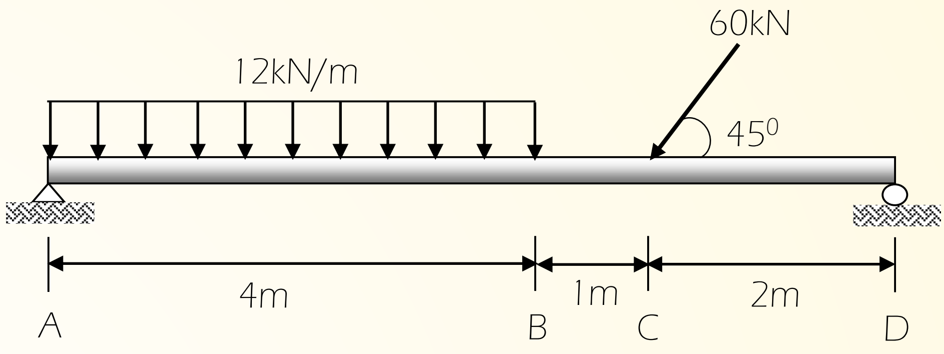


# B CANTILEVER BEAM

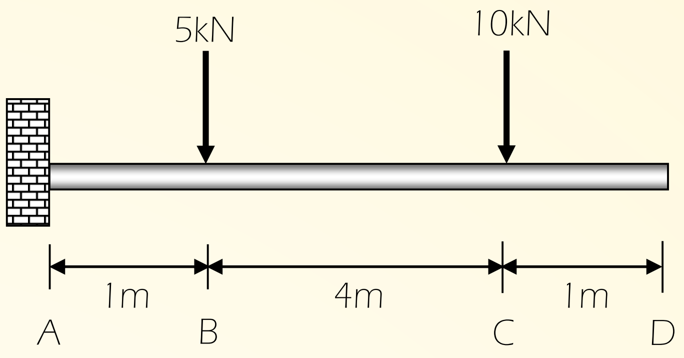
Consider a cantilever beam subjected to a load as shown in figure below.

i. Calculate the reactions at support of the beam.

ii. Draw the shear force diagram (SFD) and bending moment diagram (BMD).

iii. Hence, determine the value of maximum bending moment

Question 1

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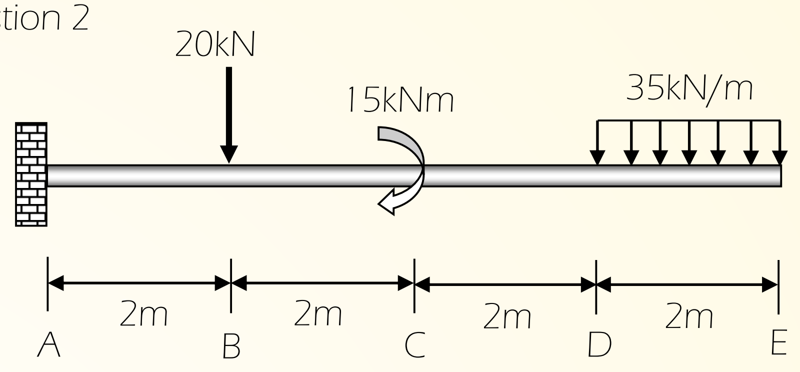

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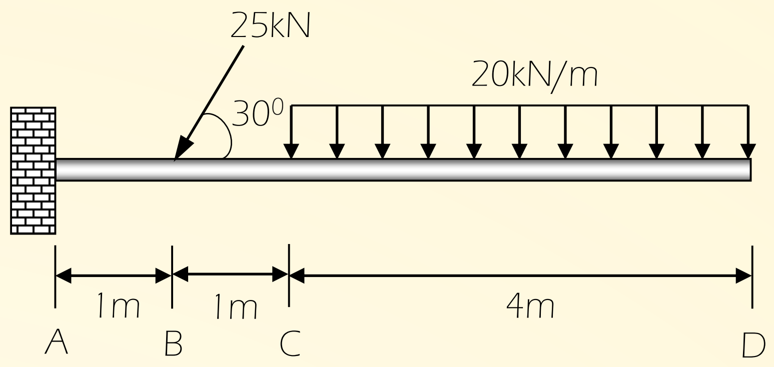


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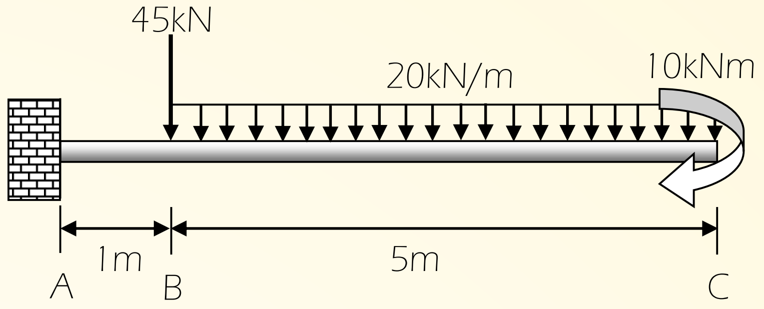
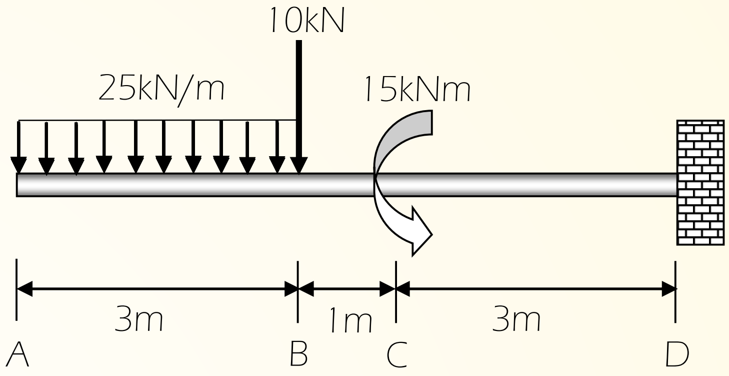
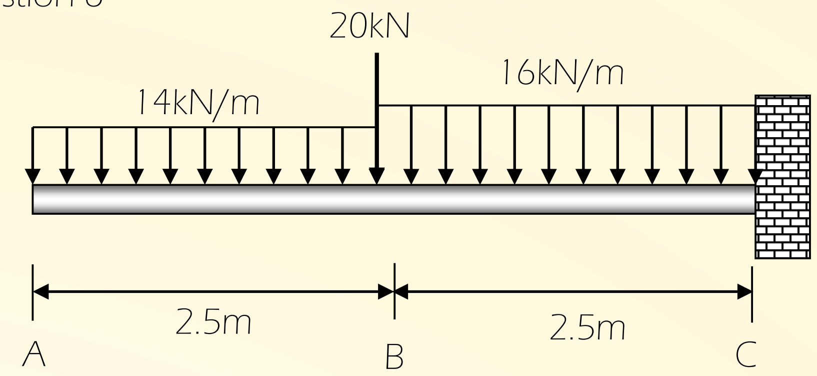


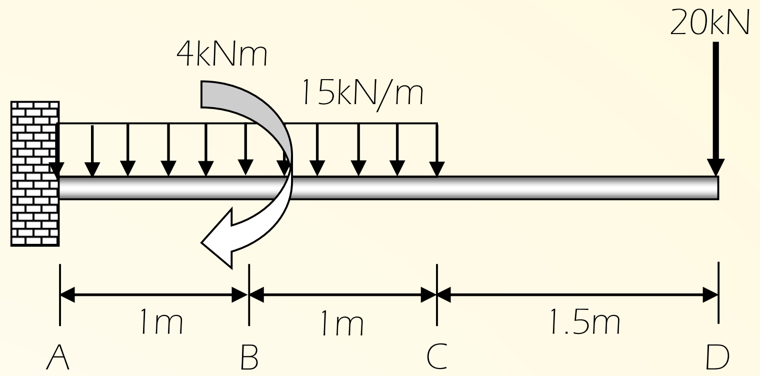

Question 5



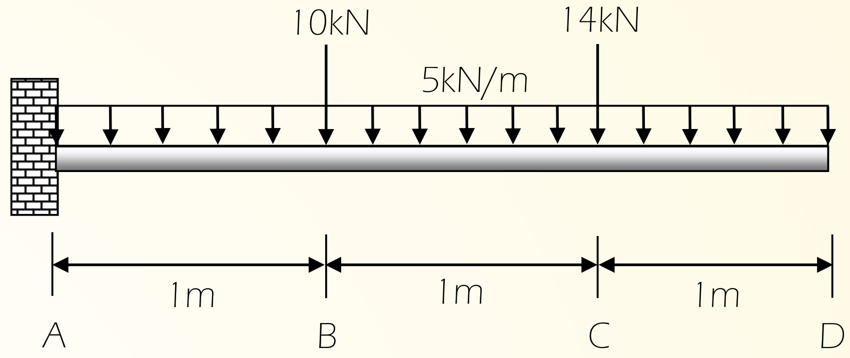
Question 6




Question 7




Question 8



# C OVERHANGING BEAM

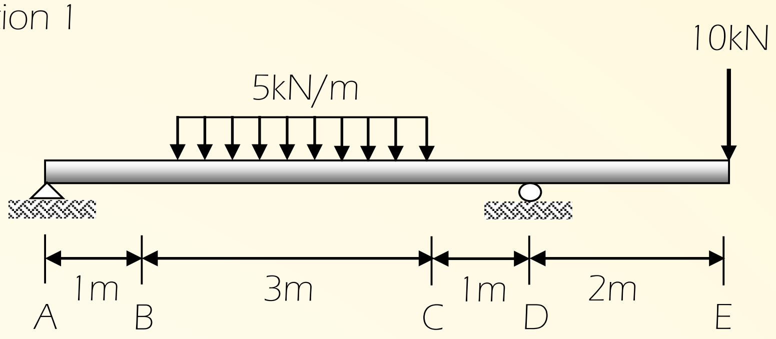
An overhanging beam is subjected to a load as shown in figure below.

i. Calculate the reactions at support of the beam.

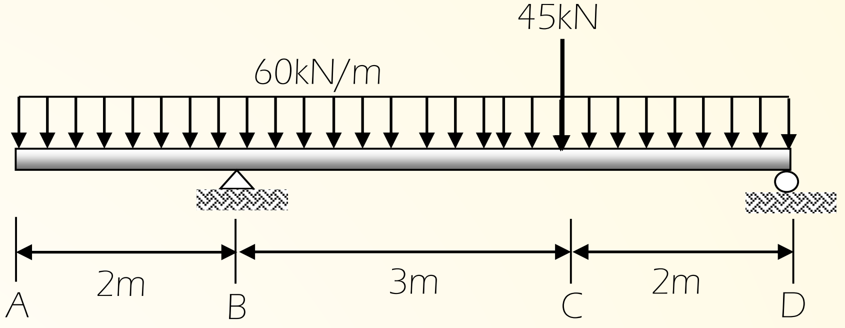
ii. Draw the shear force diagram (SFD) and bending moment diagram (BMD).

iii. Hence, determine the value of maximum bending moment and the point at which it occurs.

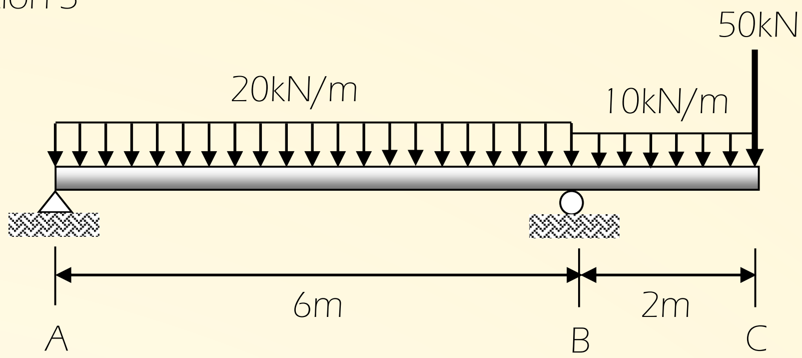
Question 1




Question 2



Question 3

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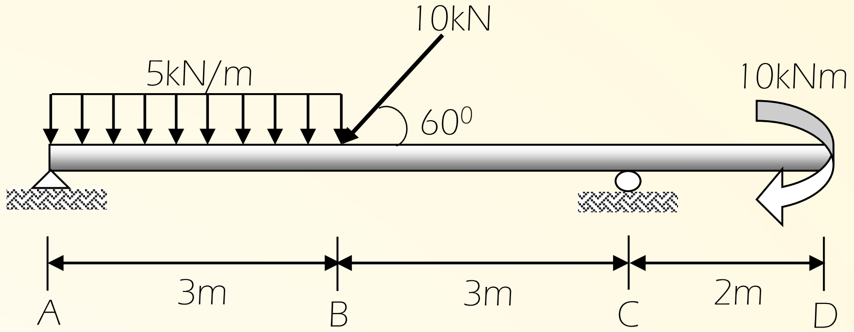
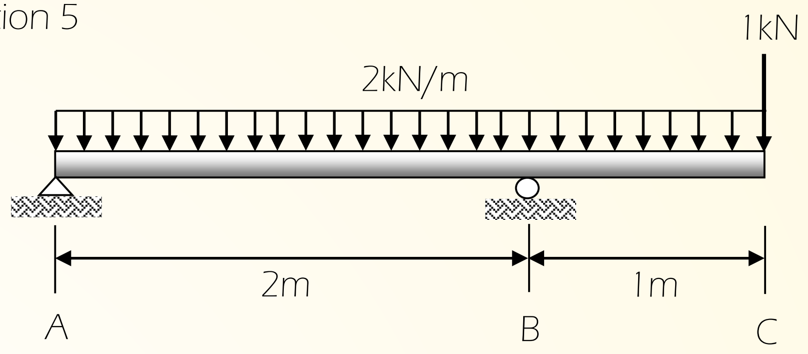
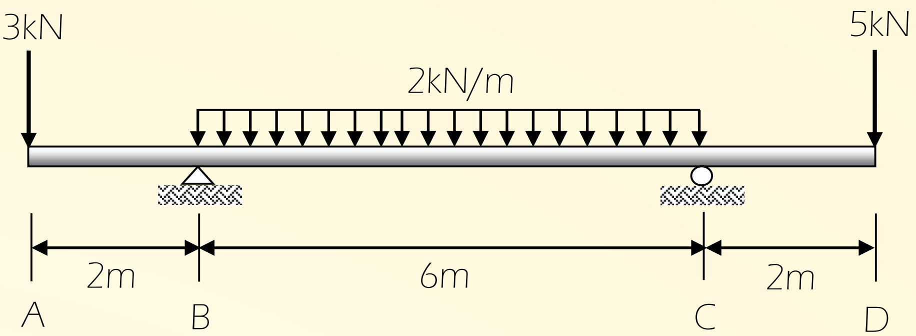



Question 5



Question 6

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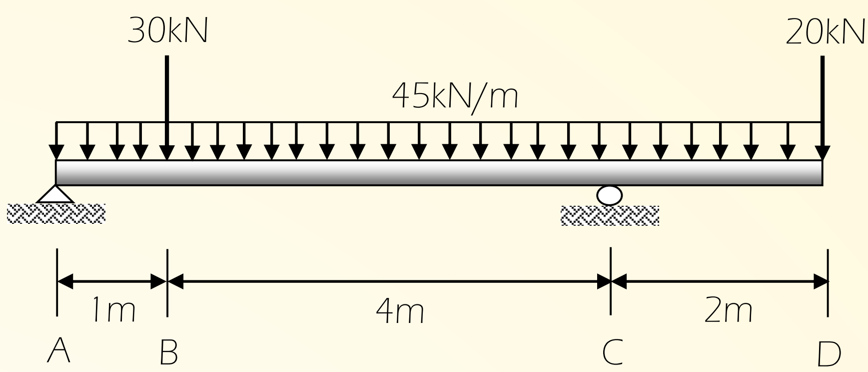
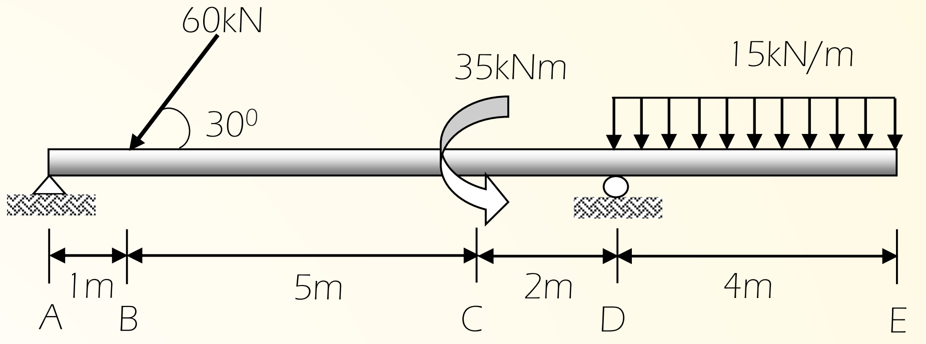



Question 8



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