SULIT

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI I : 2024/2025

BBM30073 : ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 06 JANUARI 2025
MASA : 9.00 PAGI – 12.00 TENGAH HARI (3 JAM)

Kertas ini mengandungi **LIMA (5)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** subjective questions. Answers **ALL** the questions.

ARAHAN :

*Bahagian ini mengandungi **EMPAT (4)** soalan subjektif. Jawab **SEMUA** soalan.*

QUESTION 1

SOALAN 1

CLO1 (a) Determine the solution of the following differential equation.

Tentukan penyelesaian bagi persamaan pembezaan berikut.

$$y' = 8x^3 - 4x + 1; \qquad y(1) = 3$$

[5 marks]

[*5 markah*]

CLO1 (b) Construct the differential equation for the following equation.

Bina persamaan pembezaan bagi persamaan berikut.

$$y = 3Kx^2 + 4x$$

[8 marks]

[*8 markah*]

CLO2 (c) Solve the following differential equation by using appropriate method.

Selesaikan persamaan pembezaan berikut dengan menggunakan kaedah yang sesuai.

$$x\frac{dy}{dx} = x^3 \cos 4x + 2y$$

[12 marks]

[*12 markah*]

QUESTION 2

SOALAN 2

CLO2 (a) Solve the initial value problem of the second order homogeneous differential equation.

Selesaikan masalah nilai awalan bagi persamaan pembezaan kedua.

$$y'' - 25y = 0; \quad y(0) = 2, \quad y'(0) = -5$$

[10 marks]

[*10 markah*]

CLO2 (b) Compute the following second order differential equation by using undetermined coefficients method.

Kirakan persamaan pembezaan kedua berikut dengan menggunakan kaedah pekali tak tentu.

$$\frac{d^2y}{dx^2} + 4y = 8\sin 4x$$

[15 marks]

[15 *markah*]

QUESTION 3

SOALAN 3

CLO2 (a) Solve the following second order partial differential equation by using direct partial integration.

Selesaikan persamaan perbezaan separa peringkat kedua berikut dengan menggunakan pengamiran separa langsung.

$$U_{xx} = 2x + y$$

[5 marks]

[*5 markah*]

CLO2 (b) Determine if the given equation is elliptic, hyperbolic or parabolic.

Tentukan persamaan yang diberi bersifat eliptik, hiperbolik atau parabolik.

i) $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial t} + 3\frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$

[5 marks]

[5 *markah*]

ii) $4U_{xx} + 5U_{xy} + \frac{1}{4}U_{yy} = 0$

[5 marks]

[5 *markah*]

CLO2 (c) Solve the partial differential equations with constant coefficient. Give your answer in the form of *u(x,y) = F(mx + y)*

Selesaikan persamaan pembezaan separa ini dengan Pekali Tetap. Beri jawapan anda dalam bentuk u(x,y) = F(mx + y).

$$9\frac{\partial^2 \beta}{\partial x^2} + 12\frac{\partial^2 \beta}{\partial x \partial t} + 4\frac{\partial^2 \beta}{\partial t^2} = 0$$

[10 marks]

[*10 markah*]

QUESTION 4

SOALAN 4

CLO1 (a) Determine the Laplace Transform of $f(t) = \{3e^{2t} + 4\sin 5t\}$

Tentukan Jelmaan Laplace $f(t) = \{3e^{2t} + 4\sin 5t\}$

[6 marks]

[*6 markah*]

CLO2 (b) Compute the Inverse Laplace Transform of the following equation.

Kirakan Jelmaan Laplace Songsang bagi persamaan berikut.

(i) $L^{-1}\left\{\frac{6}{s^4}\right\} + L^{-1}\left\{\frac{4}{3s^2-12s+9}\right\}$

[7 marks]

[*7 markah*]

CLO2 (c) Solve the initial value problem of the following equation.

Selesaikan masalah nilaian awal bagi persamaan berikut.

$y''(t) + 4y'(t) + 4y(t) = 6e^{-2t};\quad y(0) = -2,\ y'(0) = 8$

[12 marks]

[*12 markah*]

SOALAN TAMAT

FORMULA SHEET FOR ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY (BBM30073)

Basic Differentiation

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}[sin(ax)] = a\,cos(ax)$$

$$\frac{d}{dx}[cos(ax)] = -a\,sin(ax)$$

$$\frac{d}{dx}[tan(ax)] = a\,sec^2(ax)$$

Basic Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int udv = uv - \int vdu$$

$$\int e^{ax} du = \frac{1}{a}e^{ax} + C$$

$$\int \frac{1}{x} dx = ln|x| + C$$

$$\int sin(ax)\,dx = -\frac{1}{a}cos(ax) + C$$

$$\int cos(ax)\,dx = \frac{1}{a}sin(ax) + C$$

$$\int sec^2(ax)\,dx = \frac{1}{a}tan(ax) + C$$

First Order Differential Equation

HOMOGENOUS

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

LINEAR

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$ye^{\int P(x)\,dx} = \int Q(x) \cdot e^{\int P(x)\,dx}\,dx + C$$

EXACT

$$P(x,y)dx + Q(x,y)dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

BERNOULLI

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

$$y^{1-n}\left(e^{\int(1-n)P(x)dx}\right) = \int (1-n)Q(x)\left(e^{\int(1-n)P(x)dx}\right)dx + C$$

SEPARABLE

$\frac{dy}{dx} = f(x) \cdot g(y)$ or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$

Second Order Differential Equation

General form : $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = G(x)$	
Roots	**Form of y_c**
$m_1 \neq m_2$	$y_c = Ae^{m_1 x} + Be^{m_2 x}$
$m = m_1 = m_2$	$y_c = e^{mx}(A + Bx)$
$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $m = \propto \pm \beta i$	$y_c = e^{\propto x}(A\cos\beta x + B\sin\beta x)$

Particular Integral:

$G(x)$	$form\ of\ y_p$
$k\ (constant)$	C
kx	$Cx + D$
kx^2	$Cx^2 + Dx + E$
$k\sin \alpha x$ @ $k\cos \alpha x$	$C\cos \alpha x + D\sin \alpha x$
$k\sinh \alpha x$ @ $k\cosh \alpha x$	$C\cosh \alpha x + D\sinh \alpha x$
e^{kx}	Ce^{kx}
xe^{kx}	$(Cx + D)e^{kx}$

WRONSKIAN DETERMINANT

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

PARTICULAR INTEGRAL

$$y_p = uy_1 + vy_2$$

$$u = -\int \frac{y_2 G(x)}{W} dx \quad \text{and} \quad v = \int \frac{y_1 G(x)}{W} dx$$

Partial Differential Equation

Complementary Function:

General form : $A\frac{\partial^2 z}{\partial x^2} + B\frac{\partial^2 z}{\partial x \partial y} + C\frac{\partial^2 z}{\partial y^2} = F(x,y)$

$f\left(D_x, D_y\right) = F(x,y)$

Canonical Form	Equation	Form of Z_c
$B^2 - 4AC > 0$	Hyperbolic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$
$B^2 - 4AC = 0$	Parabolic	$Z_c = \emptyset_1(y + mx) + x\emptyset_2(y + mx)$
$B^2 - 4AC < 0$	Elliptic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$ where $m = \propto \pm \beta i$

Particular Integral:

$$\boldsymbol{Z = \frac{1}{f(D_x, D_y)}F(x,y)}$$

If $\boldsymbol{F(x,y)}$	Form of $\boldsymbol{Z_p}$
e^{ax+by}	$Z_p = \frac{1}{f(a,b)}e^{ax+by}$ if $f(a,b) \neq 0$
$sin(ax + by)$	$Z_p = \frac{1}{f(-a^2,-ab,-b^2)}sin(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$
$cos(ax + by)$	$Z_p = \frac{1}{f(-a^2,-ab,-b^2)}cos(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$

Laplace Transform Table					
No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3.	t^n $n = 1,2,3\ldots$	$\frac{n!}{s^{n+1}}$	15.	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$
7.	$t^n.e^{at}$ $n = 1,2,3\ldots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2-\omega^2}$
8.	$t^n.f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$	20.	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
9.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	21.	$\int_0^1 f(u)\,du$	$\frac{F(s)}{s}$
10.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s)-y(0)$
12.	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s)-sy(0)-y'(0)$