SULIT

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2024/2025

BBM30073: ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 16 JUN 2025
MASA : 9.00 PAGI – 12.00 T/HARI (3 JAM)

Kertas ini mengandungi **LAPAN (8)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:
This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN :
Bahagian ini mengandungi ***EMPAT (4)*** *soalan struktur. Jawab* ***SEMUA*** *soalan.*

QUESTION 1

SOALAN 1

CLO1 (a) Show that $y = Ae^{4x} + 2$ is a general solution of the differential equation for
Tunjukkan $y = Ae^{4x} + 2$ *adalah penyelesaian am bagi persamaan perbezaan bagi*

$$\frac{dy}{dx} + 8 = 4y$$

[5 marks]
[*5 markah*]

CLO1 (b) Construct a differential equation for the equation, $y = Bxe^{2x}$.
Bentukkan persamaan perbezaan bagi persamaan, $y = Bxe^{2x}$.

[7 marks]
[*7 markah*]

CLO2 (c) Solve the following first order differential equations:
Selesaikan persamaan perbezaan peringkat pertama berikut:

i) $\frac{dy}{dx} = \frac{xy}{y+1}$

[6 marks]
[*6 markah*]

ii) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2$

[7 marks]
[*7 markah*]

QUESTION 2

SOALAN 2

CLO2 (a) Given that the differential equation of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

Diberi bahawa persamaan perbezaan $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$.

i) Solve the general solution.

Selesaikan penyelesaian am.

[3 marks]

[*3 markah*]

ii) Solve the particular solution given $y(0) = 6$, $y'(0) = 2$.

Selesaikan penyelesaian khusus bagi $y(0) = 6$, $y'(0) = 2$.

[7 marks]

[*7 markah*]

CLO2 (b) Given that the equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-12y=24e^{2x}-6$. Using the method of undetermined coefficient:

Diberi bahawa persamaan $\frac{d^2y}{dx^2}+\frac{dy}{dx}-12y=24e^{2x}-6$. Gunakan *kaedah koefisien tak tentu:*

i) Solve the complementary function, y_c.
Selesaikan fungsi pelengkap, y_c.

[3 marks]
[*3 markah*]

ii) Solve the particular solution, y_p and the general solution.
Selesaikan penyelesaian khusus, y_p *dan penyelesaian amnya.*

[4 marks]
[*4 markah*]

iii) Solve the general solution when the boundary conditions are given as $y(0)=\frac{3}{2}$ and $y'(0)=14$.
Selesaikan penyelesaian am apabila syarat sempadan adalah $y(0)=\frac{3}{2}$ *dan* $y'(0)=14$.

[8 marks]
[*8 markah*]

QUESTION 3

SOALAN 3

CLO2 (a) Solve the following second order partial differential equation by using direct partial integration.

Selesaikan persamaan perbezaan separa peringkat kedua berikut dengan menggunakan pengamiran separa langsung.

$$\frac{\partial^2 u}{\partial x \partial y} = 2ylnx + 6xy^2$$

[5 marks]

[*5 markah*]

CLO2 (b) Solve the complementary function for all following partial differential equation into its canonical form.

Selesaikan fungsi pelengkap bagi semua persamaan perbezaan separa berikut dalam bentuk kanoniknya.

i) $U_{xx} + 4U_{xy} + 4U_{yy} = 0$

[3 marks]

[*3 markah*]

ii) $5\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

[3 marks]

[*3 markah*]

iii) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} = 0$

[4 marks]

[*4 markah*]

CLO2 (c) Given that the function of second order partial differential equation with constant coefficients is $\left(D_x^2 - 2D_xD_y + D_y^2\right)Z = e^{x+3y}$.

Diberi bahawa fungsi berikut bagi persamaan pembezaan separa terbitan kedua dengan pekali malar ialah $\left(D_x^2 - 2D_xD_y + D_y^2\right)Z = e^{x+3y}$.

i) Solve the complimentary function (C.F)
Selesaikan fungsi pelengkap (F.P)

[4 marks]
[*4 markah*]

ii) Solve a particular integral (P.I) and its general solution
Selesaikan penyelesaian kursus (P.K) dan penyelesaian amnya

[6 marks]
[6 *markah*]

QUESTION 4

SOALAN 4

CLO1 (a) Based on the Laplace transform table, solve the Laplace transform for:

Berdasarkan jadual jelmaan Laplace, selesaikan jelmaan Laplace bagi:

i) $\mathcal{L}(e^{-3t} + 2t - 4)$

[3 marks]

[*3 markah*]

ii) $\mathcal{L}\{(t^5 e^{7t} + tsint)\}$

[4 marks]

[*4 markah*]

CLO2 (b) Solve the inverse Laplace transform for the following functions:

Selesaikan songsangan jelmaan Laplace bagi fungsi yang berikut:

i) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+16} + \frac{6}{s^3}\right\}$

[3 marks]

[*3 markah*]

ii) $\mathcal{L}^{-1}\left\{\frac{3s+4}{(s-2)^2}\right\}$

[5 marks]

[*5 markah*]

CLO 2 (c) Consider the following differential equation:

Pertimbangkan persamaan perbezaan berikut:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 4$$

i) Solve $Y(s)$ by applying the Laplace transform and using partial fraction decomposition to simplify the expression, given the initial conditions $y(0) = 0$ and $y'(0) = 1$.

Selesaikan $Y(s)$ *dengan menggunakan transformasi Laplace dan penguraian pecahan separa untuk memudahkan ungkapan tersebut, berdasarkan syarat awal* $y(0) = 0$ *dan* $y'(0) = 1$.

[7 marks]
[*7 markah*]

ii) Solve $y(t)$ by using the inverse Laplace transform of $Y(s)$.

Selesaikan $y(t)$ *dengan menggunakan transformasi songsang Laplace ke atas* $Y(s)$.

[3 marks]
[*3 markah*]

SOALAN TAMAT

FORMULA

Basic Differentiation	Basic Integration
$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1$
$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\int u dv=uv-\int v du$
$\frac{d}{dx}(e^{ax})=ae^{ax}$	$\int e^{ax}du=\frac{1}{a}e^{ax}+C$
$\frac{d}{dx}(ln\lvert x\rvert)=\frac{1}{x}$	$\int \frac{1}{x}dx=ln\lvert x\rvert+C$
$\frac{d}{dx}[sin(ax)]=a\,cos(ax)$	$\int sin(ax)\,dx=-\frac{1}{a}cos(ax)+C$
$\frac{d}{dx}[cos(ax)]=-a\,sin(ax)$	$\int cos(ax)\,dx=\frac{1}{a}sin(ax)+C$
$\frac{d}{dx}[tan(ax)]=a\,sec^2(ax)$	$\int sec^2(ax)\,dx=\frac{1}{a}tan(ax)+C$

First Order Differential Equation

HOMOGENOUS $y=vx$ $\frac{dy}{dx}=v+x\frac{dy}{dx}$	LINEAR $\frac{dy}{dx}+P(x)y=Q(x)$ $ye^{\int P(x)\,dx}=\int Q(x)\cdot e^{\int P(x)\,dx}\,dx+C$
EXACT $P(x,y)dx+Q(x,y)dy=0$ $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$	BERNOULLI $\frac{dy}{dx}+P(x)y=Q(x)y^n$ $y^{1-n}\left(e^{\int(1-n)P(x)dx}\right)=\int(1-n)Q(x)\left(e^{\int(1-n)P(x)dx}\right)dx+C$

Second Order Differential Equation

General form : $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=G(x)$

Roots	Form of y_c
$m_1 \neq m_2$	$y_c=Ae^{m_1x}+Be^{m_2x}$
$m=m_1=m_2$	$y_c=e^{mx}(A+Bx)$
$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $m=\propto\pm\beta i$	$y_c=e^{\propto x}(Acos\beta x+Bsin\beta x)$

Second Order Differential Equation

Particular Integral:

$G(x)$	$form\ of\ y_p$
$k\ (constant)$	C
kx	$Cx + D$
kx^2	$Cx^2 + Dx + E$
$k\sin\alpha x$ @ $k\cos\alpha x$	$C\cos\alpha x + D\sin\alpha x$
$k\sinh\alpha x$ @ $k\cosh\alpha x$	$C\cosh\alpha x + D\sinh\alpha x$
e^{kx}	Ce^{kx}
xe^{kx}	$(Cx + D)e^{kx}$

WRONSKIAN DETERMINANT

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$$

PARTICULAR INTEGRAL

$$y_p = uy_1 + vy_2$$

$$u = -\int \frac{y_2G(x)}{W}dx \quad \text{and} \quad v = \int \frac{y_1G(x)}{W}dx$$

Partial Differential Equation

Complementary Function:

General form : $A\frac{\partial^2 z}{\partial x^2} + B\frac{\partial^2 z}{\partial x\partial y} + C\frac{\partial^2 z}{\partial y^2} = F(x, y)$

$f(D_x, D_y) = F(x, y)$

Canonical Form	Equation	Form of Z_c
$B^2 - 4AC > 0$	Hyperbolic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$
$B^2 - 4AC = 0$	Parabolic	$Z_c = \emptyset_1(y + mx) + x\emptyset_2(y + mx)$
$B^2 - 4AC < 0$	Elliptic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$ where $m = \alpha \pm \beta i$

Particular Integral:

$$Z_p = \frac{1}{f(D_x, D_y)}F(x, y)$$

If $F(x, y)$	Form of Z_p
e^{ax+by}	$Z_p = \frac{1}{f(a,b)}e^{ax+by}$ if $f(a, b) \neq 0$
$\sin(ax + by)$	$Z_p = \frac{1}{f(-a^2,-ab,-b^2)}\sin(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$
$\cos(ax + by)$	$Z_p = \frac{1}{f(-a^2,-ab,-b^2)}\cos(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$

Laplace Transform Table					
No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3.	t^n $n = 1,2,3\ldots$	$\frac{n!}{s^{n+1}}$	15.	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$
7.	$t^n . e^{at}$ $n = 1,2,3\ldots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2-\omega^2}$
8.	$t^n . f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$	20.	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
9.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	21.	$\int_0^1 f(u)\, du$	$\frac{F(s)}{s}$
10.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s) - y(0)$
12.	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s) - sy(0) - y'(0)$