SULIT

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2024/2025

BBM30093: PROBABILITY AND STATISTICS FOR ENGINEERING TECHNOLOGY

TARIKH : 18 JUN 2025
MASA : 9.00 PAGI – 12.00 T/HARI (3 JAM)

Kertas ini mengandungi **SEPULUH (10)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Kertas Graf, Formula dan Jadual.

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN:

*Bahagian ini mengandungi **EMPAT (4)** soalan berstruktur. Jawab **SEMUA** soalan.*

QUESTION 1
SOALAN 1

CLO1 (a) i. Explain the application of statistics in the engineering field with one example.
Terangkan aplikasi statistik dalam bidang kejuruteraan dengan satu contoh.

[2 marks]
[*2 markah*]

ii. Classify each of the following as qualitative or quantitative data:
Klasifikasikan setiap data berikut sebagai data kualitatif atau kuantitatif:

- Student ID number / *Nombor ID pelajar*
- Height of students / *Ketinggian pelajar*
- Favourite color / *Warna kegemaran.*

[3 marks]
[*3 markah*]

CLO1 (b) A manufacturing company records the daily production output (in units) of its six production lines over a month. The data is grouped into the following frequency distribution in Table 1(b).
Sebuah syarikat pembuatan merekodkan hasil pengeluaran harian (dalam unit) bagi enam barisan pengeluarannya sepanjang sebulan. Data tersebut dikelompokkan ke dalam taburan kekerapan seperti dalam Jadual 1(b).

Table 1(b) / *Jadual 1(b)*

Production Output (Units) *Hasil Pengeluaran (Unit)*	Frequency (Days) *Frekuensi (Hari)*
50-59	4
60-69	6
70-79	8
80-89	10
90-99	2
100-109	5

i. Construct a histogram to represent the frequency distribution using the data provided in Table 1(b).
Bina histogram untuk mewakili taburan kekerapan data yang disediakan dalam Jadual 1(b).

[5 marks]
[*5 markah*]

ii. If the company aims to maintain a minimum daily production of 75 units, calculate the percentage of days that meet or exceed this target.
Jika syarikat bermatlamat mengekalkan pengeluaran harian minimum sebanyak 75 unit, hitung peratusan hari yang mencapai atau melebihi sasaran ini.

[5 marks]
[*5 markah*]

CLO2 (c) A factory operates six machines, each taking a different amount of times (in minutes) to complete a task. The recorded processing times are:
Sebuah kilang mengendalikan enam mesin, setiap satu mengambil masa yang berbeza (dalam minit) untuk menyelesaikan satu tugas. Masa pemprosesan yang direkodkan adalah seperti berikut:

$$10, 12, 15, x, 20, 22$$

i. If the mean processing time for all six machines is 15 minutes, calculate the value of x where x is an unknown processing time.
Jika min masa pemprosesan bagi semua enam mesin ialah 15 minit, hitung nilai x dimana x adalah masa pemprosesan yang tidak diketahui.

[3 marks]
[*3 markah*]

ii. Calculate the mean deviation of the processing times.
Kira sisihan min bagi masa pemprosesan.

[3 marks]
[*3 markah*]

iii. Calculate the variance and standard deviation of the processing times.
*Kira varians dan sisihan piawai bagi masa pemprosesa*n.

[4 marks]
[*4 markah*]

QUESTION 2

SOALAN 2

CLO1 (a) i. A factory produces light bulbs, and each bulb is tested for quality. The result of each test can be either defective (D) or non-defective (N). Express the sample space for testing two bulbs selected at random.

Sebuah kilang menghasilkan mentol lampu, dan setiap mentol diuji untuk kualiti. Hasil setiap ujian sama ada rosak (D) atau tidak rosak (N). Nyatakan ruang sampel bagi ujian dua mentol yang dipilih secara rawak.

[2 marks]

[*2 markah*]

ii. Explain the characteristics of normal distribution.

Terangkan ciri- ciri taburan normal.

[3 marks]

[*3 markah*]

CLO2 (b) A quality control engineer at a factory inspects three randomly selected machines. Each machine can either be functional (F) or faulty (X). Based on historical data, the probability of a machine being faulty is 0.2, and the probability of a machine being functional is 0.8. Calculate,

Seorang jurutera kawalan kualiti di sebuah kilang memeriksa tiga mesin yang dipilih secara rawak. Setiap mesin sama ada berfungsi (F) atau rosak (X). Berdasarkan data lepas ,kebarangkalian sebuah mesin rosak ialah 0.2 dan kebarangkalian sebuah mesin berfungsi ialah 0.8. Kira,

i. the probability that all three machines are functional.

kebarangkalian bahawa ketiga-tiga mesin berfungsi.

[3 marks]

[*3 markah*]

ii. the probability that exactly one machine is faulty.
kebarangkalian bahawa tepat satu mesin rosak.

[3 marks]
[*3 markah*]

iii. the probability that at least one machine is faulty.
kebarangkalian bahawa sekurang-kurangnya satu mesin rosak.

[4 marks]
[*4 markah*]

CLO2

(c) The speed of vehicles on a particular highway is normally distributed with a mean of 110 km/h and a standard deviation of 12 km/h. Calculate the probability that a randomly selected car is traveling with the following velocity:
Kelajuan kenderaan di lebuh raya tertentu mengikut taburan normal dengan min 110 km/j dan sisihan piawai 12 km/j. Hitung kebarangkalian bahawa sebuah kereta yang dipilih secara rawak mempunyai kelajuan yang berikut:

i. more than 130 km/h.
lebih daripada 130 km/h.

[5 marks]
[5 *markah*]

ii. between 90 km/h and 130 km/h.
antara 90 km/j dan 130 km/j.

[5 marks]
[5 *markah*]

QUESTION 3

SOALAN 3

CLO2 (a) A random sample of a size 100 is taken from a population and the data is summarized as follows:

Sampel rawak bersaiz 100 diambil daripada populasi dan data diringkaskan seperti berikut:.

$$\Sigma x = 1000 \quad , \quad \Sigma\,(x-\bar{x})^2 = 170.8$$

i. Calculate the point estimate and the 95% symmetrical confidence interval for the population mean.

Kirakan titik anggaran dan *95% selang keyakinan simetri untuk min populasi.*

[10 marks]

[*10 markah*]

ii. Calculate the 99% symmetrical confidence interval for the population mean.

Kirakan 99% selang keyakinan simetri untuk min populasi.

[5 marks]

[*5 markah*]

CLO2 (b) A supervisor wants to know the proportion of defective electronic components produced by a machine. He inspected a random sample of 200 electronic components and found that 40 electronic components are defective.

Seorang penyelia ingin mengetahui bahagian komponen elektronik yang rosak yang dihasilkan oleh mesin. Beliau telah memeriksa sampel rawak 200 komponen elektronik dan mendapati 40 komponen elektronik adalah rosak.

i. Calculate the proportion of defective electronic components, $\hat{p}$ and $\hat{q}$.

Kira perkadaran komponen elektronik yang rosak, $\hat{p}$ dan $\hat{q}$.

[5 marks]

[5 *markah*]

ii. Calculate the 95% confidence interval of the proportion of defective electronic components produced by the machine.
Cari selang keyakinan 95% untuk perkadaran komponen elektronik yang rosak yang dihasilkan oleh mesin.

[5 *marks*]
[5 *markah*]

QUESTION 4

SOALAN 4

CLO1 (a) A university administrator claims that the average study hours per week for students are at least 20 hours. A researcher wants to test this claim and collects data from a random sample of 29 students. The sample has an average study time of 18.5 hours per week with a standard deviation of 4.2 hours. Assume that the study hours follow normal distribution.

Seorang pentadbir universiti mendakwa bahawa purata masa belajar pelajar dalam seminggu adalah sekurang-kurangnya 20 jam. Seorang penyelidik ingin menguji dakwaan ini dan mengumpulkan data daripada sampel rawak 29 pelajar. Sampel tersebut mempunyai purata masa belajar sebanyak 18.5 jam seminggu dengan sisihan piawai 4.2 jam. Andaikan bahawa masa belajar mengikut taburan normal.

i. Express the null and alternative hypotheses for this test.

Nyatakan hipotesis nol dan hipotesis alternatif untuk ujian ini.

[2 marks]

[2 *markah*]

ii. Explain why a t-test is appropriate for this scenario?

Terangkan mengapa ujian-t sesuai digunakan dalam situasi ini?

[3 marks]

[3 *markah*]

CLO2 (b) A manufacturer claims that the bulbs produced by their company will last more than 3000 hours. A sample of 50 bulbs is taken and the sample mean is found to be 3100 hours, and the sample standard deviation is 900 hours. By assuming that the populations are normally distributed, conduct a hypothesis test at the 5% significance level.

Seorang pengilang mendakwa bahawa mentol yang dihasilkan oleh syarikat mereka akan bertahan lebih daripada 3000 jam. Sampel 50 mentol diambil dan min sampel didapati adalah 3100 jam dan sampel sisihan piawai adalah 900 *jam. Dengan mengandaikan bahawa populasi mengikut taburan normal, jalankan ujian hipotesis pada aras signifikan 5%.*

[10 marks]

[*10 markah*]

CLO2 (c) A study is conducted to determine if Company A retains its workers longer than Company B and the data is compared in Table 4(c). Assuming that the populations are normally distributed, conduct a hypothesis test at the 5% significance level. (Assume σ^2 is unknown but equal).

Satu kajian dijalankan untuk menentukan sama ada Syarikat A mengekalkan pekerjanya lebih lama daripada Syarikat B dan data dibandingkan dalam Jadual 4(c). Dengan mengandaikan bahawa populasi mengikut taburan normal, jalankan ujian hipotesis pada aras signifikan 5%. (*Andaikan σ^2 tidak ketahui tetapi sama*).

Table 4(c) / *Jadual 4(c)*

	Company A *Syarikat A*	Company B *Syarikat B*
Sample Mean, $\bar{X}$ *Min sampel,* $\bar{X}$	5 years /*tahun*	4.5 years/ *tahun*
Standard deviation, s *Sisihan piawai,* s	1.2 years /*tahun*	0.8 *years* / *tahun*
Number of sample, n *Bilangan sampel,* n	15	20

[10 marks]

[*10 markah*]

SOALAN TAMAT

FORMULA SHEET FOR PROBABILITY & STATISTICS FOR ENGINEERING TECHNOLOGY BBM30093

DESCRIPTIVE STATISTICS

Ungrouped Data	Grouped Data
Mean, $\overline{X} = \frac{\sum X}{n}$	Mean, $\overline{X} = \frac{\sum f.X_m}{n}$
Mean Deviation = $E = \frac{\Sigma\lvert x-\bar{x}\rvert}{n}$	Population Variance, $\sigma^2 = \frac{\sum f(X-\mu)^2}{N}$
Population Variance, $\sigma^2 = \frac{\sum(X-\mu)^2}{N}$	Sample Variance, $s^2 = \frac{\sum f\left(X-\overline{X}\right)^2}{n-1}$
Sample Variance, $s^2 = \frac{\sum\left(X-\overline{X}\right)^2}{n-1}$	$or \ \ s^2 = \frac{\sum f{X_m}^2-\left[\frac{(\sum fX_m)^2}{n}\right]}{n-1}$
$or \ \ s^2 = \frac{\sum X^2-\left[\frac{(\sum X)^2}{n}\right]}{n-1}$	Median, $M = L_M + \left(\frac{\frac{N}{2}-F}{f_M}\right)C$
Population Standard Deviation, $\sigma = \sqrt{\sigma^2}$	Mode, $M_o = L_{M_o} + \left(\frac{d_1}{d_1+d_2}\right)C$
Sample Standard Deviation, $s = \sqrt{s^2}$	

PROBABILITY & STATISTICS

Addition Rule (mutually exclusive events), $P(A \cup B) = P(A) + P(B)$	Conditional Probability, $P(B\lvert A) = \frac{P(A\cap B)}{P(A)}$
Addition Rule (events not mutually exclusive), $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	Complementary events, $P\left(\overline{A}\right) = 1 - P(A)$
Multiplication Rule (Independent event), $P(A \cap B) = P(A) \cdot P(B)$	Permutation Rule, ${}_nP_r = \frac{n!}{(n-r)!}$
Multiplication Rule(dependent event), $P(A \cap B) = P(A).P(B\lvert A)$	Combination Rule, ${}_nC_r = \frac{n!}{(n-r)!r!}$
Mean for a probability distribution, $\mu = \sum[X \cdot P(X)]$	Normal distribution Standard score, $z = \frac{X-\mu}{\sigma}$ or $\frac{X-\overline{X}}{s}$
Variance for a probability distribution. $\sigma^2 = \sum[X^2.P(X)] - \mu^2$	Mean of sample mean, $\mu_{\overline{X}} = \mu$
Standard deviation for a probability distribution. $\sigma = \sqrt{\sum[X^2.P(X)] - \mu^2}$	Standard error of the means, $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$
Expectation of Discrete Random Variable, $E(X) = \sum[X.P(X)]$	Central limit theorem formula, $z = \frac{X-\mu}{\sigma/\sqrt{n}}$
	Binomial probability, $P(X) = \frac{n!}{(n-X)!\,X!} \cdot p^x \cdot q^{n-x}$
	Mean for binomial distribution, $\mu = np$

Variance of Discrete Random Variable

$$Var(X) = E(X^2) - [E(X)]^2$$

Where $E(X^2) = \sum x^2 . P(X = x)$

Variance and standard deviation for the binomial distribution, $\sigma^2 = npq \quad \sigma = \sqrt{npq}$

<u>SAMPLING AND ESTIMATION</u>

z confidence interval for means,

$$\overline{X} - z\alpha/_2\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \overline{X} + z\alpha/_2\left(\frac{\sigma}{\sqrt{n}}\right)$$

t confidence interval for means,

$$\overline{X} - t\alpha/_2\left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t\alpha/_2\left(\frac{s}{\sqrt{n}}\right)$$

Sample size for means, $n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$,

Confidence interval for a proportion,

$$\hat{p} - (z_{\alpha/2})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\alpha/2})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Sample size for proportion, $n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$,

Confidence interval for variance,

$$\frac{(n-1)s^2}{\chi^2_{right}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{left}}$$

Confidence interval for standard deviation,

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{right}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{left}}}$$

<u>HYPOTHESIS TESTING</u>

Test for the population mean:

z test, $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$, variance known

z test, $Z = \frac{\bar{X}-\mu}{s/\sqrt{n}}$, variance unknown

t test, $t = \frac{\bar{X}-\mu}{s/\sqrt{n}}$,small sample

Variance is known (Large or small sample)

Test statistics, $z = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$

Variance is unknown (Large sample)

Test statistics, $z = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$

Test for two population mean:

Variance is unknown but equal

Test statistics, $t = \frac{(\bar{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{s_{p^2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

Pool sample variance, $s_{p^2} = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Degree of freedom, $df = n_1 + n_2 - 2$

Variance is unknown but unequal

Test statistics, $t = \frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s^2}{n_1}+\frac{s_2^2}{n_2}}}$

Degree of freedom, $df = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$

Table B–1 The Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

For z values greater than 3.49, use 0.4999.

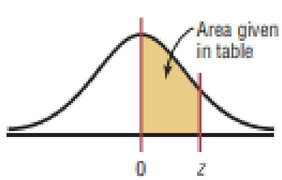


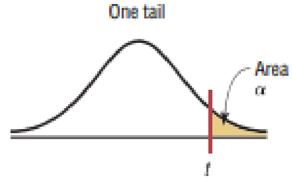
Table F The *t* Distribution

d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.045	2.462	2.756
30		1.310	1.697	2.042	2.457	2.750
32		1.309	1.694	2.037	2.449	2.738
34		1.307	1.691	2.032	2.441	2.728
36		1.306	1.688	2.028	2.434	2.719
38		1.304	1.686	2.024	2.429	2.712
40		1.303	1.684	2.021	2.423	2.704
45		1.301	1.679	2.014	2.412	2.690
50		1.299	1.676	2.009	2.403	2.678
55		1.297	1.673	2.004	2.396	2.668
60		1.296	1.671	2.000	2.390	2.660
65		1.295	1.669	1.997	2.385	2.654
70		1.294	1.667	1.994	2.381	2.648
75		1.293	1.665	1.992	2.377	2.643
80		1.292	1.664	1.990	2.374	2.639
90		1.291	1.662	1.987	2.368	2.632
100		1.290	1.660	1.984	2.364	2.626
500		1.283	1.648	1.965	2.334	2.586
1000		1.282	1.646	1.962	2.330	2.581
(z) ∞		1.282[a]	1.645[b]	1.960	2.326[c]	2.576[d]

[a] This value has been rounded to 1.28 in the textbook.
[b] This value has been rounded to 1.65 in the textbook.
[c] This value has been rounded to 2.33 in the textbook.
[d] This value has been rounded to 2.58 in the textbook.

Source: Adapted from W. H. Beyer, *Handbook of Tables for Probability and Statistics*, 2nd ed., CRC Press, Boca Raton, Fla., 1986. Reprinted with permission.

One tail



Two tails

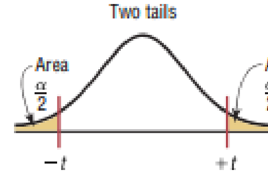


Table G The Chi-Square Distribution

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169