SULIT

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2024/2025

BBM30143: ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 16 JUN 2025
MASA : 9.00 PAGI – 12.00 T/HARI (3 JAM)

Kertas ini mengandungi **LAPAN (8)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** subjective questions. Answer ALL questions.

ARAHAN :

Bahagian ini mengandungi ***EMPAT (4)*** *soalan subjektif. Jawab SEMUA soalan*

QUESTION 1

SOALAN 1

CLO1 (a) Solve the initial value problem for the following differential equation.

Selesaikan masalah nilai awalan bagi persamaan pembezaan berikut:

$$y' = 2x^3y^2, \quad y(1) = 2$$

[5 marks]

[*5 markah*]

CLO1 (b) Construct the differential equation for the following equation.

Bina sebuah persamaan pembezaan bagi persamaan berikut.

$$y = Ae^{3x} + Bx + 5$$

[7 marks]

[7 *markah*]

CLO2 (c) Solve the following differential equations by using appropriate method.

Selesaikan persamaan pembezaan berikut dengan mengunakan kaedah yang sesuai.

i) $\dfrac{dy}{dx} = \dfrac{3xy}{y^2 + 1}$

[6 marks]

[6 *markah*]

ii) $x\dfrac{dy}{dx} - 2y = \sqrt{x}$

[7 marks]

[7 *markah*]

QUESTION 2

SOALAN 2

CLO2 (a) Given the second order differential equation as follows.

Diberikan persamaan pembezaan kedua seperti berikut.

$$y'' - 6y' + 9y = 0$$

i) Construct the general solution of the second order homogenous differential equation.

Bina persamaan umum bagi persamaan pembezaan homogen peringkat kedua.

[3 marks]

[3 markah]

ii) Solve the second order differential equation if given $y(0) = 2, y'(0) = 1$.

Selesaikan persamaan pembezaan kedua jika diberi $y(0) = 2,\ y'(0) = 1$.

[7 marks]

[7 markah]

CLO2 (b) Given the second order differential equation.

Diberikan persamaan pembezaan peringkat kedua.

$$y'' - 5y' - 6y = 5$$

i) Calculate the complementry solution, y_c.

Kirakan penyelesaian pelengkap, y_c.

[3 marks]

[3 markah]

ii) Determine the Wronskian (W) based on the finding in b) i).
Tentukan nilai Wronskian (W) *berdasarkan jawapan b) i).*

[4 marks]
[4 markah]

iii) By using answer in b) i) and b) ii), compute its general solution.
Dengan menggunakan jawapan dari b) i) dan b) ii), hitungkan penyelesaian umumnya.

[8 marks]
[8 markah]

QUESTION 3
SOALAN 3

CLO2 (a) Solve the general solution for second order partial differential equation.
Selesaikan persamaan umum bagi persamaan pembezaan separa peringkat kedua.

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{3x^2 y + y^2}{xy} + \cos 2x$$

[5 marks]
[5 markah]

CLO2 (b) Show the following partial differential equations as either hyperbolic, parabolic or elliptic.
Tunjukkan persamaan pembezaan separa berikut sama ada persamaan hiperbola, parabola atau elips.

i) $$\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial x \partial y} + 25\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

[3 marks]
[3 markah]

ii) $$\frac{\partial^2 u}{\partial x^2} - 12\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} - 5\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0$$

[3 marks]
[3 markah]

iii) $$3\frac{\partial^2 f}{\partial x^2} = 5\frac{\partial^2 f}{\partial y^2}$$

[4 marks]
[4 markah]

CLO2

(c) Construct the solution of second order partial differential equations with constant coefficient in the form of $u(x,y) = F(mx + y)$.
Bina penyelesaian bagi persamaan pembezaan separa kedua berikut dalam bentuk $u(x,y) = F(mx + y)$.

i) $U_{xx} - 4U_{xy} + 5U_{yy} = 0$

[4 marks]
[4 markah]

ii) $4U_{yy} = 12U_{xy} - 9U_{xx}$

[6 marks]
[6 markah]

QUESTION 4

SOALAN 4

CLO1 (a) Determine the Laplace Transform for each of the following functions by using the Laplace Transform Table.

Tentukan jelmaan Laplace bagi setiap fungsi berikut dengan menggunakan Jadual Jelmaan Laplace.

i) $f(t) = \frac{1}{2}(t^2 + 5e^{2t})$

[3 marks]

[3 markah]

ii) $f(t) = \cos 3t + 2t \sin 5t$

[4 marks]

[4 markah]

CLO2 (b) Solve the following inverse Laplace Transform.

Selesaikan jelmaan Laplace sonsangan berikut:

i) $L^{-1}\left\{\frac{(5-s)^2}{s^4}\right\}$

[3 marks]

[3 markah]

ii) $L^{-1}\left\{\frac{4s}{s^2+9} - \frac{3}{s^4}\right\}$

[5 marks]

[5 markah]

CLO2

(c) i) Solve the initial value problem by using Laplace Transform.

Selesaikan masalah nilai awalan dengan menggunakan Jelmaan Laplace.

$y' + 3y = 2e^{-t}$, $y(0) = 0$

[7 marks]

[7 markah]

ii) Write the expression of $2y'' - 3y' + y$ in Laplace Transform if given $y(0) = 2$, $y'(0) = -3$.

Tuliskan ungkapan $2y'' - 3y' + y$ *dalam bentuk Jelmaan Laplace jika diberi* $y(0) = 2$, $y'(0) = -3$.

[3 marks]

[3 markah]

SOALAN TAMAT

FORMULA FOR ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY BBM30143

Basic Differentiation	Basic Integration
$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\int udv = uv - \int vdu$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}du = \frac{1}{a}e^{ax} + C$
$\frac{d}{dx}(ln\|x\|) = \frac{1}{x}$	$\int \frac{1}{x}dx = ln\|x\| + C$
$\frac{d}{dx}[sin(ax)] = a\,cos(ax)$	$\int sin(ax)\,dx = -\frac{1}{a}cos(ax) + C$
$\frac{d}{dx}[cos(ax)] = -a\,sin(ax)$	$\int cos(ax)\,dx = \frac{1}{a}sin(ax) + C$
$\frac{d}{dx}[tan(ax)] = a\,sec^2(ax)$	$\int sec^2(ax)\,dx = \frac{1}{a}tan(ax) + C$

First Order Differential Equation

METHOD	FORMULA
Separable	$\frac{dy}{dx} = f(x) \cdot g(y)$ or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$
Homogenous	$y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$
Linear	$\frac{dy}{dx} + P(x)y = Q(x)$ $ye^{\int P(x)\,dx} = \int Q(x) \cdot e^{\int P(x)\,dx}\,dx + C$
Bernoulli	$\frac{dy}{dx} + P(x)y = Q(x)y^n$ $y^{1-n}\left(e^{\int(1-n)P(x)dx}\right) = \int (1-n)Q(x)\left(e^{\int(1-n)P(x)dx}\right)dx + C$
Exact	$P(x,y)dx + Q(x,y)dy = 0$; P and Q have same degree. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

Second Order Differential Equation

General form of second order differential: $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = G(x)$

Quadratic Equation: $m = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Characteristic Equation: $am^2 + bm + c = 0$

Complementary Function:

Types	**Roots**	**Form of y_c**
Different Real Roots $b^2 - 4ac > 0$	$m_1 \neq m_2$	$y_c = Ae^{m_1x} + Be^{m_2x}$
Equal Real Roots $b^2 - 4ac = 0$	$m = m_1 = m_2$	$y_c = e^{mx}(A + Bx)$
Complex Roots $b^2 - 4ac < 0$	$m = \propto \pm \beta i$	$y_c = e^{\propto x}(Acos\beta x + Bsin\beta x)$

Particular Integral:

$G(x)$	$form\ of\ y_p$
$k\ (constant)$	C
kx	$Cx + D$
kx^2	$Cx^2 + Dx + E$
$ksin\ \alpha x$ @ $kcos\ \alpha x$	$Ccos\ \alpha x + Dsin\ \alpha x$
$ksinh\ \alpha x$ @ $kcosh\ \alpha x$	$Ccosh\ \alpha x + Dsinh\ \alpha x$
e^{kx}	Ce^{kx}
xe^{kx}	$(Cx + D)e^{kx}$
x^2e^{kx}	$(Cx^2 + Dx + E)e^{kx}$

Wronskian Determinant

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$$

Particular Integral

$$y_p = uy_1 + vy_2$$

$$u = -\int \frac{y_2G(x)}{W}dx \qquad and \qquad v = \int \frac{y_1G(x)}{W}dx$$

Partial Differential Equation

General form of second order partial differential:

$$A\frac{\partial^2 z}{\partial x^2} + B\frac{\partial^2 z}{\partial x \partial y} + C\frac{\partial^2 z}{\partial y^2} = F(x,y)$$

$$f(D_x, D_y) = F(x,y)$$

Complementary Function:

Canonical Form	Equation	Form of Z_c
$B^2 - 4AC > 0$	Hyperbolic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$
$B^2 - 4AC = 0$	Parabolic	$Z_c = \emptyset_1(y + mx) + x\emptyset_2(y + mx)$
$B^2 - 4AC < 0$	Elliptic	$Z_c = \emptyset_1(y + m_1x) + \emptyset_2(y + m_2x)$ where $m = \propto \pm \beta i$

Particular Integral:

$$\boldsymbol{Z = \frac{1}{f(D_x, D_y)} F(x,y)}$$

If $F(x,y)$	Form of Z_p
e^{ax+by}	$Z_p = \frac{1}{f(a,b)} e^{ax+by}$ if $f(a,b) \neq 0$
$sin(ax + by)$	$Z_p = \frac{1}{f(-a^2, -ab, -b^2)} sin(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$
$cos(ax + by)$	$Z_p = \frac{1}{f(-a^2, -ab, -b^2)} cos(ax + by)$ if $f(-a^2, -ab, -b^2) \neq 0$

Laplace Transform Table

No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3.	$t^n, n = 1,2,3\ldots$	$\frac{n!}{s^{n+1}}$	15.	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at}\sinh\omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at}\sinh\omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$
7.	$t^n.e^{at}n = 1,2,3\ldots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at}\cosh\omega t$	$\frac{s+a}{(s+a)^2-\omega^2}$
8.	$t^n.f(t)$	$(-1)^n\frac{d^n}{ds^n}[F(s)]$	20.	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
9.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	21.	$\int_0^1 f(u)\,du$	$\frac{F(s)}{s}$
10.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s)-y(0)$
12.	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s)-sy(0)-y'(0)$