STRUCTURED (100 Marks)

INSTRUCTION :

This section consists of **SIX (6)** structured question. Anwer **FOUR (4)** questions only.

QUESTION 1

a) Find the values of the following functions:

i. $\cosh(3.5)$ (2 marks)

ii. $\text{cosech}\sqrt{2.5}$ (3 marks)

iii. $\dfrac{\sinh 2}{\cosh(-0.8)}$ (3 marks)

b) $v^2 = 0.45L\tanh\left[\dfrac{5.5d}{L}\right]$ is a formula for velocity of waves over the bottom of shallow water, where d is the depth and L is the wavelength. If $d = 7.5m$ and $L = 105m$, calculate the value of v.

(4 marks)

c) Using relevant sketches, determine the principle value for the following:

i. $\sin^{-1}(0.523)$ (4 marks)

ii. $\cos^{-1}(-0.457)$ (4 marks)

d) Prove that $\cosh 2x = 2\sinh^2 x + 1$. (5 marks)

POLITEKNIK

Jabatan Pengajian Politeknik

EXAMINATION AND EVALUATION DIVISION
DEPARTMENT OF POLYTECHNIC EDUCATION
(MINISTRY OF HIGHER EDUCATION)

MATHEMATICS, SCIENCE & COMPUTER DEPARTMENT

FINAL EXAMINATION

DECEMBER 2011 SESSION

B5001 : ENGINEERING MATHEMATICS 5

DATE : 25 APRIL 2012 (WEDNESDAY)
DURATION : 2 HOURS (8.30 AM – 10.30 AM)

This paper consists of **NINE (9)** pages including the front page and appendix.

This paper consists of **SIX (6)** questions.
Answer **FOUR (4)** questions only.

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DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE CHIEF INVIGILATOR

QUESTION 3

a) If $z = \tan 2y - 2x^2y^3$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (3 marks)

b) If $z = (4x - 2y)(3x + 5y)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (4 marks)

c) If $z = \frac{y^2}{x^3} + 2$, find $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$. (5 marks)

d) If $z = \ln(y^2 + x^2)$, find $\frac{\partial z}{\partial y}$ and $\frac{\partial^2 z}{\partial y^2}$. (4 marks)

e) The power P dissipated in a resistor is given by $P = \frac{E^2}{R}$. If the value of $E = 80V$ and $R = 9\Omega$, find the change in P resulting from a drop of $0.9V$ in E and an increase of $0.3\ \Omega$ in R.

(9 marks)

QUESTION 2

a) Differentiate the following functions with respect to x:

i. $y = \ln(\sinh x)$ (3 marks)

ii. $y = \sinh^{-1}(\sinh x)$ (5 marks)

iii. $y = \tan^{-1}(\sinh x)$ (5 marks)

b) Show that $\frac{d}{dx}\left\{\sin^{-1}\left(\frac{x}{a}\right)\right\} = \frac{1}{\sqrt{a^2-x^2}}$. Hence find $\frac{d}{dx}\left\{\sin^{-1}\left(\frac{x}{2}\right)\right\}$.

(7 marks)

c) Find the derivative for $x^3y + y^3x - 4\ln xy = 5$. (5 marks)

QUESTION 5

a) Evaluate the following using the integration by parts method:

i. $\int x\, e^{-2x}\, dx$ (4 marks)

ii. $\int e^{3x} \sin x \; dx$ (7 marks)

b) Evaluate the following integrals by using partial fractions:

i. $\int \frac{3x-1}{x^2 - x - 2} dx$ (9 marks)

ii. $\int \frac{4x^2 + x + 1}{x^2 - x} dx$ (5 marks)

QUESTION 4

a) Integrate the following functions with respect to x:

i. $\int \frac{dx}{\sqrt{1-x^2}}$ (3 marks)

ii. $\int \frac{x}{\sqrt{x^4 - 3}} dx$ (5 marks)

iii. $\int \frac{dx}{x^2 + 12x + 11}$ (5 marks)

iv. $\int_{-3}^{0} \frac{dx}{x^2 + 6x + 13}$ (7 marks)

b) Solve the following:

$\int_{0}^{1} \sinh^2 x \; dx$ (5 marks)

B 5001 ENGINEERING MATHEMATICS FORMULA

TRIGONOMETRIC IDENTITIES

$\cos^2 x + \sin^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\operatorname{cosec}^2 x = 1 + \cot^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

HYPERBOLIC IDENTITIES

$\cosh^2 x - \sinh^2 x = 1$

$\operatorname{sech}^2 x = 1 - \tanh^2 x$

$\operatorname{cosech}^2 x = \coth^2 x - 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 1 + 2\sinh^2 x$

$= 2\cosh^2 x - 1$

$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

HIPERBOLIC FUNCTIONS

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$\operatorname{cosech} x = \dfrac{2}{e^x - e^{-x}}, \quad x \neq 0$

$\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$

$\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}, \quad x \neq 0$

INVERSE HIPERBOLIC FUNCTIONS

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right), \; -\infty < x < \infty$

$\cosh^{-1} x = \pm\ln\left(x + \sqrt{x^2 - 1}\right), \; x \geq 1$

$\tanh^{-1} x = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}, \quad |x| < 1$

$\operatorname{sech}^{-1} x = \ln\left[\dfrac{1 + \sqrt{1 - x^2}}{x}\right], \; 0 < x \leq 1$

$\operatorname{cosech}^{-1} x = \ln\left[\dfrac{1}{x} + \dfrac{\sqrt{1 + x^2}}{|x|}\right], \quad x \neq 0$

$\coth^{-1} x = \dfrac{1}{2}\ln\dfrac{x+1}{x-1}, \quad |x| > 1$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

$\dfrac{d}{dx}(\sin u) = \cos u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\sin u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan u) = \sec^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec u) = \sec u \tan u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u \cot u \dfrac{du}{dx}$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS

$\dfrac{d}{dx}(\sinh u) = \cosh u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\cosh u) = \sinh u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u \coth u \dfrac{du}{dx}$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS

$\dfrac{d}{dx}(\sinh^{-1} u) = \dfrac{1}{\sqrt{1+u^2}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cosh^{-1} u) = \dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tanh^{-1} u) = \dfrac{1}{1-u^2}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\coth^{-1} u) = \dfrac{-1}{u^2-1}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{sech}^{-1} u) = \dfrac{-1}{u\sqrt{1-u^2}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{cosech}^{-1} u) = \dfrac{-1}{|u|\sqrt{1+u^2}}\dfrac{du}{dx}$

DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTIONS

$\dfrac{d}{dx}(\sin^{-1} u) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos^{-1} u) = \dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan^{-1} u) = \dfrac{1}{1+u^2}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cot^{-1} u) = \dfrac{-1}{1+u^2}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec^{-1} u) = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{cosec}^{-1} u) = \dfrac{-1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

QUESTION 6

a) Find the differential equation for the function below:

$y = x^2 + Ax$ (4 marks)

b) Solve the following first order differential equations:

i. $\dfrac{dy}{dx} = 5x^2 - 12x + 25$ (4 marks)

ii. $\dfrac{dy}{dx} = \dfrac{3x}{y+2}$ (4 marks)

iii. $\dfrac{dy}{dx} - \dfrac{5}{2}y = e^{-\frac{3}{2}x}$ (5 marks)

c) Determine the general solution for the following:

i. $2\dfrac{d^2y}{dx^2} + 12\dfrac{dy}{dx} + 16y = 0$ (4 marks)

ii. $\dfrac{d^2y}{dx^2} + 2\sqrt{2}\dfrac{dy}{dx} + 2y = 0$ (4 marks)

INTEGRATION OF HYPERBOLIC FUNCTIONS	**INTEGRATION OF INVERSE FUNCTIONS**	
$\int \sinh x\, dx = \cosh x + c$ $\int \cosh x\, dx = \sinh x + c$ $\int \operatorname{sech}^2 x\, dx = \tanh x + c$ $\int \operatorname{cosech}^2 x\, dx = -\coth x + c$ $\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + c$ $\int \operatorname{cosech} x \coth x\, dx = -\operatorname{cosech} x + c$	$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + c$ $\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}\frac{u}{a} + c$ $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$ $\int \frac{-du}{a^2 + u^2} = \frac{1}{a}\cot^{-1}\frac{u}{a} + c$ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left\|\frac{u}{a}\right\| + c$ $\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{cosec}^{-1}\left\|\frac{u}{a}\right\| + c$	$\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\frac{u}{a} + c$ $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\frac{u}{a} + c$ $\int \frac{du}{a^2 - u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a} + c,$ $\int \frac{du}{u^2 - a^2} = -\frac{1}{a}\coth^{-1}\frac{u}{a} + c,$ $\int \frac{du}{\|u\|\sqrt{a^2 + u^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left\|\frac{u}{a}\right\| + c$ $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a} + c$