

SECTION A

INSTRUCTION:

This section consists of **TWO (2)** questions (**QUESTION 1 and QUESTION 2**).

Answer **ONE OF** the questions.

QUESTION 1

- a) In the expansion of $\left(\frac{3}{2}x + 3\right)^5$, find: [CLO 1]
- the first four terms. (5 marks)
 - the coefficient independent of x . (7 marks)
- b) Expand the function $\frac{x+1}{(2+4x)^5}$ up to the first four terms. [CLO 1]
- (6 marks)
- c) Expand $\left(1 - \frac{x}{2}\right)^7$ by using the Binomial Theorem in ascending power of x including the term in x^3 . Then, find the value of $(0.88)^7$ correct to 3 decimal places.
- [CLO 1]
- (7 marks)

EXAMINATION AND EVALUATION DIVISION
DEPARTMENT OF POLYTECHNIC EDUCATION
(MINISTRY OF HIGHER EDUCATION)

CIVIL ENGINEERING DEPARTMENT

FINAL EXAMINATION
DECEMBER 2011 SESSION

BA501 : ENGINEERING MATHEMATICS 4

DATE : 25 APRIL 2012 (WEDNESDAY)
DURATION : 2 HOURS (11.15 AM – 1.15 PM)

This paper consists of **TWELVE (12)** pages including the front page and appendix.

This paper consists of **EIGHT(8)** questions. Answer **FOUR (4)** questions only.

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THE CHIEF INVIGILATOR

(CLO stated at the end of each question is referring to the learning outcome of the topic assessed. The CLO stated is only for lectures' references.)

SECTION B

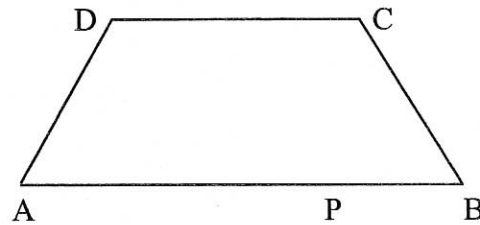
INSTRUCTION:

This section consists of TWO (2) questions (QUESTION 3 and QUESTION 4).

Answer ONE OF the questions.

QUESTION 3

a)



ABCD is a trapezium where AB is parallel to DC and $AB = \frac{4}{3}DC$. P is a point on AB line, where $AP = 3PB$. Given that $\overrightarrow{AB} = 4\hat{a}$, find the following vector in terms of \hat{a} . [CLO 2]

i) \overrightarrow{AP} (1 mark)

ii) \overrightarrow{DC} (1 mark)

iii) $\overrightarrow{AP} + \overrightarrow{DC}$ (1 mark)

b) If $\overrightarrow{OP} = a\hat{i} + 4\hat{j}$ and $\overrightarrow{OQ} = \hat{i} - 2\hat{j}$, given $|\overrightarrow{OP}| = 5$, find: [CLO 2]

i. a (2 marks)

ii. angle between \overrightarrow{OP} and \overrightarrow{OQ} (4 marks)

QUESTION 2

a) Expand the first four terms for the following power series. [CLO 1]

$$(3x - 1)\ln\left(1 + \frac{x}{3}\right) \quad (5 \text{ marks})$$

b) Find the coefficient of x^3 in the expansion of $\frac{e^{(x-3)}}{3}$. [CLO 1] (5 marks)

c) Using Taylor series, expand the first four terms for $f(x) = \frac{1}{3x-1}$, at $x_0 = 2$. [CLO 1] (7 marks)

d) Using Mc Laurin series, expand the first four terms for $f(x) = 3e^{-4x} + \frac{2}{e^x}$. [CLO 1] (8 marks)

QUESTION 4

a) Construct the partial fraction of the following: [CLO 2]

i) $\frac{x-1}{(3x-5)(x-3)}$ (5 marks)

ii) $\frac{1}{(x-3)(x+1)^2}$ (8 marks)

b) Convert $\frac{x^3+16}{x^3-4x^2+8x}$ to proper fraction. Then, find the partial fraction.

[CLO 2]

(12 marks)

c) Given $\vec{OA} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{OC} = 3\hat{i} + \hat{j} - 3\hat{k}$ are position vectors.

Find: [CLO 2]

i) direction cosine of \vec{AB} (5 marks)

ii) $\vec{OA} \times (\vec{OB} \times \vec{OC})$ (4 marks)

iii) $\vec{OA} \cdot (\vec{OB} \times \vec{OC})$ (3 marks)

iv) $\vec{OC} \cdot (\vec{OA} \times \vec{OB})$ (4 marks)

QUESTION 6

a) Find the inverse Laplace transform for the following functions. [CLO 3]

i. $F(s) = \frac{24}{s^5} + \frac{2s}{s^2+36} - \frac{7}{s+7}$ (5 marks)

ii. $F(s) = \frac{4s+5}{s^2+9} + \frac{4}{s^2}$ (6 marks)

b) Find the inverse Laplace transform using partial fraction method for the following function. [CLO 3]

$$\frac{3s+11}{(s-3)(s+2)} \quad (7 \text{ marks})$$

c) Sketch a graph for the following functions. [CLO 3]

$$u(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 2, & 2 \leq t < 5 \\ 1, & 5 \leq t \end{cases} \quad (3 \text{ marks})$$

d) Write the following functions in terms of unit step function and find the Laplace transform of the functions. [CLO 3]

$$f(t) = \begin{cases} 4, & 0 \leq t < 5 \\ 1, & 5 \leq t \end{cases} \quad (4 \text{ marks})$$

SECTION C

INSTRUCTION:

This section consists of TWO (2) questions (QUESTION 5 and QUESTION 6).

Answer ONE OF the questions.

QUESTION 5

a) Using the formula, derive the Laplace transform of the following functions.

[CLO 3]

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

i. $f(t) = 16$ (5 marks)

ii. $f(t) = 2e^{6t}$ (5 marks)

b) Find the Laplace transform for the following functions using the Laplace transform Table. [CLO 3]

i. $f(t) = t^3 + 8t^5 - \sin 5t$ (3 marks)

ii. $f(t) = 10 + 2e^{3t} - 3 \cos 4t - 5t^4$ (4 marks)

iii. $f(t) = e^{3t} \cosh 2t + 3e^{-4t} \sin 6t - 5$ (4 marks)

iv. $f(t) = 8 \sin t + 5e^{-5t} \sinh 3t - 4e^{-7t} \cos 8t$ (4 marks)

QUESTION 8

- a) Find the focus and directrix of the following parabolic equation. Then, sketch the graph. [CLO 4]

i) $y^2 = 16x$ (3 marks)

ii) $x^2 = -8y$ (3 marks)

- b) Find the vertex, focus point, eccentric and directrix for the following ellipse equation. Then, sketch the curve. [CLO 4]

$$\frac{x^2}{36} + \frac{y^2}{81} = 1 \quad (9 \text{ marks})$$

- c) Find the tangent and normal equation at point (6,-1) for the following hyperbolic equation. [CLO 4]

$$x^2 - 12y^2 = 32 \quad (10 \text{ marks})$$

SECTION D

INSTRUCTION:

This section consists of TWO (2) questions (QUESTION 7 and QUESTION 8).

Answer ONE OF the questions.

QUESTION 7

- a) Sketch $f(x) = e^x$ and $f(x) = e^{-x}$. Then, find the point of intersection for both functions. $(-2 \leq x \leq 2)$. [CLO 3]

(6 marks)

- b) Find the equation for each of the circle which has [CLO 3]

i) Center (2,-1) and radius 3. (4 marks)

ii) Center (-3,7) and tangent to the x-axis. (5 marks)

- c) State the center and radius of each of the following circles. [CLO 3]

i) $x^2 + y^2 + 2y - 2x = 4$ (5 marks)

ii) $3x^2 + 3y^2 - 3x - 6y - 2 = 0$ (5 marks)

Parabola

1.	Vertical	i. $x^2 = 4ay$	ii. $(x-h)^2 = 4a(y-k)$
2.	Horizontal	i. $y^2 = 4ax$	ii. $(y-k)^2 = 4a(x-h)$
3.	Vertex	$v = (h, k)$	
4.	Focus	$(h+a, k)$ - horizontal	$(h, k+a)$ - vertical
5.	Directrix	i. $x = h-a$	ii. $y = k-a$

Ellipse

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbola

1. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ horizontal
 2. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ vertical

Laplace Transform

NUM	$f(t)$	$F(s)$			
1	a	$\frac{a}{s}$	9	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
2	at	$\frac{a}{s^2}$	10	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
3	e^{-at}	$\frac{1}{s+a}$	11	$\sinh \omega t$	$\frac{\omega}{(s^2 - \omega^2)}$
4	te^{-at}	$\frac{1}{(s+a)^2}$	12	$\cosh \omega t$	$\frac{s}{(s^2 - \omega^2)}$
5	t^n	$\frac{n!}{s^{n+1}}$	13	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
6	$\sin \omega t$	$\frac{\omega}{(s^2 + \omega^2)}$	14	$\frac{df}{dt}$	$sF(s) - f(0)$
7	$\cos \omega t$	$\frac{s}{(s^2 + \omega^2)}$	15	$\int_0^t f(u) du$	$\frac{F(s)}{s}$
8	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16	$f(t-a)u(t-a)$	$e^{-as}F(s)$
			17	$t^n \cdot f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$

Trigonometric Identities

1	$\sin 2x = 2 \sin x \cos x$
2	$\cos 2x = 2 \cos^2 x - 1 = 1 - \sin^2 x$

FORMULA OF ENGINEERING MATHEMATICS 4 (BA501)

Binomial Expansion

1.	$(a+x)^n = a^n + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + \dots + x^n$	(n = positive integer)
2.	$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$	(n = negative integer or fraction)

Power Series

1.	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!}$
2.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n}$
3.	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^{(n)}(0)x^n}{n!}$ (MACLAURIN)
4.	$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!} + \dots + \frac{f^{(n)}(x_0)(x-x_0)^n}{n!}$ (TAYLOR)

Vector and Scalar

1.	$\vec{A} \cdot \vec{B} = a_1 a_2 + b_1 b_2 + c_1 c_2$	3.	$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{ \vec{A} \vec{B} }$	5.	Direction Cosine \vec{OP} $\cos \alpha = \frac{x}{ \vec{OP} }$ $\cos \beta = \frac{y}{ \vec{OP} }$ $\cos \gamma = \frac{z}{ \vec{OP} }$
2.	$\vec{A} \times \vec{B} = \begin{pmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$	4.	Unit vector $\hat{u} = \frac{\vec{u}}{ \vec{u} }$	6.	Area of a triangle $\frac{1}{2} \vec{AB} \times \vec{BC} $

Non Linear Equation (Circle)

1.	$(x-a)^2 + (y-b)^2 = r^2$
2.	$x^2 + y^2 + 2gx + 2fy + c = 0$ $r = \sqrt{g^2 + f^2 - c}$ center = $(-g, -f)$
3.	Equation of a tangent, $y - y_1 = m(x - x_1)$