SULIT

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENGAJIAN TINGGI

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI II : 2021 / 2022

BBM30073 : ADVANCED CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 27 JUN 2022
MASA : 9.00 PAGI - 12.00 TENGAH HARI (3 JAM)

Kertas ini mengandungi **EMPAT (4)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:
This section consists of **FOUR (4)** subjective questions. Write your answers in the answer sheet form provided.
ARAHAN :
*Bahagian ini mengandungi **EMPAT (4)** soalan subjektif. Tulis jawapan anda di dalam helaian kertas yang disediakan.*

QUESTION 1 (30 marks)

SOALAN 1 (30 markah)

CLO 1
C3

a) Solve $\frac{dy}{dx}e^{-x} + e^{4x} = 0$ by using an appropriate method. Given $y(0) = 3$.

Selesaikan $\frac{dy}{dx}e^{-x} + e^{4x} = 0$ *menggunakan kaedah yang sesuai.*

Diberi $y(0) = 3$. [10 marks]

[10 *markah*]

CLO 1
C3

b) Solve the general solution of the differential equation for each of the following functions:

Selesaikan penyelesaian umum bagi persamaan pembezaan bagi fungsi berikut:

i) $y = B\sin 3x + C\cos 3x$

[4 marks]

[4 *markah*]

ii) $y = Ax^2 + Bx$

[6 marks]

[6 *markah*]

CLO 2
C3

c) Solve the solution of the differential equation below by using an appropriate method.

Selesaikan persamaan pembezaan di bawah dengan menggunakan kaedah yang sesuai.

$$y' = \frac{3xy+y}{3x^2}$$

[10 marks]

[10 *markah*]

QUESTION 2 (20 marks)

SOALAN 2 (20 markah)

CLO 2
C3

a) Solve the second order of differential equations $y'' + 6y' - 16y = 7y' - 10y$
Selesaikan persamaan pembezaan peringkat kedua $y'' + 6y' - 16y = 7y' - 10y$ [6 marks]
[6 *markah*]

CLO 2
C3

b) Solve the non-homogenous equation using a variation of the parameter method.
Selesaikan persamaan bukan homogen menggunakan kaedah variasi parameter.

$$y'' - y' = e^{5x}$$ [14 marks]
[14 *markah*]

QUESTION 3 (20 marks)

SOALAN 3 (20 markah)

CLO 2
C3

a) Solve the general solution for second partial differential equation below:
Selesaikan penyelesaian umum bagi persamaan separa kedua di bawah:

i. $\frac{\partial^2 u}{\partial x^2} = 5x - y$ [4 marks]
[4 *markah*]

ii. $\frac{\partial^2 u}{\partial x \partial y} = 5sin3x + 2cosy$ [4 marks]
[4 *markah*]

CLO 2
C3

b) Show the solution for each of the following second-order partial differential equations and give your answer in the form of $U(x, y) = f(mx + y)$.
Tunjukkan penyelesaian bagi setiap persamaan separa peringkat kedua di bawah dan berikan jawapan dalam bentuk $U(x, y) = f(mx + y)$.

i. $4U_{xx} - 7U_{xy} + 3U_{yy} = 0$ [5 marks]
[5 *markah*]

ii. $3\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$ [7 marks]
[7 *markah*]

QUESTION 4 (30 marks)

SOALAN 4 (30 markah)

CLO 1
C3

a) Calculate the functions below by using the Theorem of Multiplication with tn:

Kirakan fungsi di bawah menggunakan Teorem Pendaraban dengan tn:

$$f(t) = t^2 e^{-8t}$$

[7 marks]

[7 markah]

CLO 2
C3

b) Solve the Inverse Laplace Transform for the following expressions by using Partial Fraction Method:

Selesaikan Jelmaan Laplace Songsangbagi ungkapan berikut menggunakan Kaedah Pecahan Separa:

$$F(s) = \frac{s^2 + 3}{(s+2)(s-1)}$$

[8 marks]

[8 markah]

CLO 2
C3

c) Solve the given initial value problem using the Laplace transform for the equation below:

Selesaikan masalah nilai awal yang diberi dengan menggunakan Jelmaan Laplace bagi persamaan di bawah :

$$y'' + 8y = 2,\ y(0) = 2\ ;\ y'(0) = 3$$

[15 marks]

[15 *markah*]

SOALAN TAMAT

FORMULA

Basic Differentiation	Basic Integration
$\frac{dy}{dx}=\frac{du}{dx}v+u\frac{dv}{dx}$	$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq -1$
$\frac{dy}{dx}=\frac{\frac{du}{dx}v-u\frac{dv}{dx}}{v^2}$	$\int u\,dv=uv-\int v\,du$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,du=\frac{1}{a}e^{ax}+C$
$\frac{d}{dx}\left(\ln\lvert x\rvert\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\ln\lvert x\rvert+C$
$\frac{d}{dx}\left[\sin(ax)\right]=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+C$
$\frac{d}{dx}\left[\cos(ax)\right]=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+C$
$\frac{d}{dx}\left[\tan(ax)\right]=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+C$

First Order Differential

Separable: $\frac{dy}{dx}=f(x)\cdot g(y)$	Linear: $\frac{dy}{dx}+P(x)y=Q(x)$; $ye^{\int P(x)\,dx}=\int Q(x)\cdot e^{\int P(x)\,dx}dx+c$
Homogeneous: $P(x,y)dx+Q(x,y)dy=0$, P and Q have same degree.	Bernoulli: $\frac{dy}{dx}+P(x)y=Q(x)y^n$
Exact: $P(x,y)dx+Q(x,y)dy=0$, $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$	$y^{1-n}e^{\int(1-n)P(x)\,dx}=\int(1-n)\cdot Q(x)\cdot e^{\int(1-n)P(x)\,dx}dx$

Second Order Differential

Quadratic Equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Roots real and different $m=m_1$ and $m=m_2$

$\therefore$ Solution is $y=Ae^{m_1x}+Be^{m_2x}$

Roots real and equal $m_1 = m_2$

$\therefore$ Solution is $y = e^{m_1 x}(A + Bx)$

Complex roots $m = \alpha \pm \beta i$

$\therefore$ Solution is $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$$\frac{d^2y}{dx^2} + n^2 y = 0 \qquad m = \pm n_i$$

$\therefore$ Solution is $y = A\cos nx + B\sin nx$

$$\frac{d^2y}{dx^2} - n^2 y = 0 \qquad m = \pm n$$

$\therefore$ Solution is $y = A\cosh nx + B\sinh nx$

Particular Integral

If $G(x)$	Assume (y_p)
k (constant)	A
kx	$Ax + B$
kx^2	$Ax^2 + Bx + C$
$k\sin\alpha x$ or $k\cos\alpha x$	$A\cos\alpha x + B\sin\alpha x$
$k\sinh\alpha x$ or $k\cosh\alpha x$	$A\cosh\alpha x + B\sinh\alpha x$
e^{kx}	Ae^{kx}

Wronskian Determinant

$$W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= y_1 y_2' - y_2 y_1'$$

Particular solution

$$y_p = -y_1 \int \frac{(y_2)(G(x))}{W} dx + y_2 \int \frac{(y_1)(G(x))}{W} dx$$

Second Order Partial Differential
$b^2 - 4ac > 0$ Hyperbolic equation $b^2 - 4ac < 0$ Elliptic equation $b^2 - 4ac = 0$ Parabolic equation

Laplace Transform Table

No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
3.	$t^n,\ n=1,2,3\ldots$	$\frac{n!}{s^{n+1}}$	15.	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh\omega t$	$\frac{s}{s^2-\omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at}\sinh\omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at}\sinh\omega t$	$\frac{\omega}{(s+a)^2-\omega^2}$
7.	$t^n.\,e^{at}\quad n=1,2,3\ldots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at}\cosh\omega t$	$\frac{s+a}{(s+a)^2-\omega^2}$
8.	$t^n.f(t)$	$(-1)^n\frac{d^n}{ds^n}[F(s)]$	20.	$f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
9.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	21.	$\int_o^t f(u)du$	$\frac{F(s)}{s}$
10.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s)-y(0)$
12.	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s)-sy(0)-y'(0)$