POLITEKNIK SULTAN SALAHUDDIN ABDUL AZIZ SHAH

FLUID MECHANICS

MOMENTUM EQUATION -PROBLEM SOLVING

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ZURINA BINTI SAFEE
FARIHAH BT MANSOR

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B I B L I O G R A P H Y

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P R E F A C E

The production of this e-book is to be used as an interesting alternative learning aid. It developed to allow students to understand the easier method of solving the Momentum Equation. This topic also include in DCC30122 . This eBook presents the principles behind the methods of solving problem for determinate force when subjected to different cases.

With these responsibilities in mind, the objective for this eBook is to develop the student's ability to recognize basic knowledge of fluids mechanics. We provide an adequate number of self-test and problem solving to enhance student knowledge and understanding. Any suggestions, comments and feedback for further improvement are most welcome.

ACKNOWLEDGEMENT

First of all, thanks to Allah S.W.T because of the help, writer finished producing the eBook entitled “Momentum Equations-Problem Solving”. We would like to acknowledge the assistance and encouragement of our families and friends who have actively contributed either directly or indirectly to the completion of this eBook. We are very grateful to the Head of Civil Engineering Department and colleagues for the encouragement and support and giving us the opportunity to produce this eBook. We hope that this eBook can benefit students in increasing their understanding of the basic momentum equations of fluid mechanics. Hopefully, this eBook can help the readers to gain more knowledge about problem solving in Momentum Equation.

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CHAPTER 1

MOMENTUM EQUATION-PROBLEM SOLVING

Impact of A Jet Flat Plate

WHAT IS MOMENTUM?

PRINCIPLE OF MOMENTUM EQUATION

Conservation od momentum is another conservation law comparable to conservation of mass. Conservation of momentum uses Newton's second law of flowing fluid.

Newton's First Law of Motion

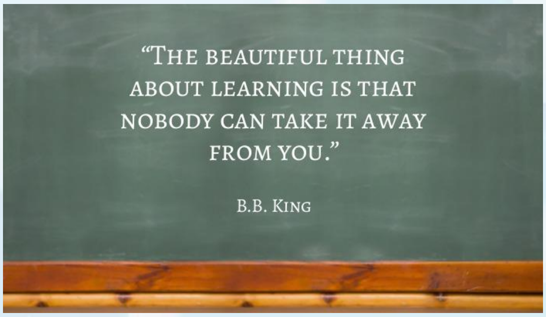
Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

Newton’s second Law of Motion

Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

Newton’s Third Law of Motion

For every action there is an equal and opposite reaction.



Derivation Of Momentum Equation in Fluid

REFRESHMENT

From Newton's second law on motion,

$$Momentum = \frac{mv}{\Delta t}$$

Since, Mass is constant, velocity will change due to time, then:

$$Momentum = m\,[\frac{v2 - v1}{\Delta t}]$$

From equation of,

Density, $\rho = \frac{mass}{Volume}$........(1)

Discharge, $Q = \frac{Volume}{time}$.......(2)

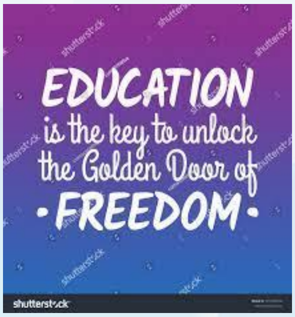
Therefore; (1) x(2)

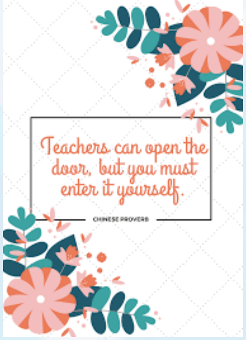
$$\frac{mass}{Volume} x \frac{Volume}{time}$$

$$\frac{m}{t} = \rho Q$$

If ρ and Q does not change,

Hence, $\Sigma F = \rho Q(v_2 - v_1)$

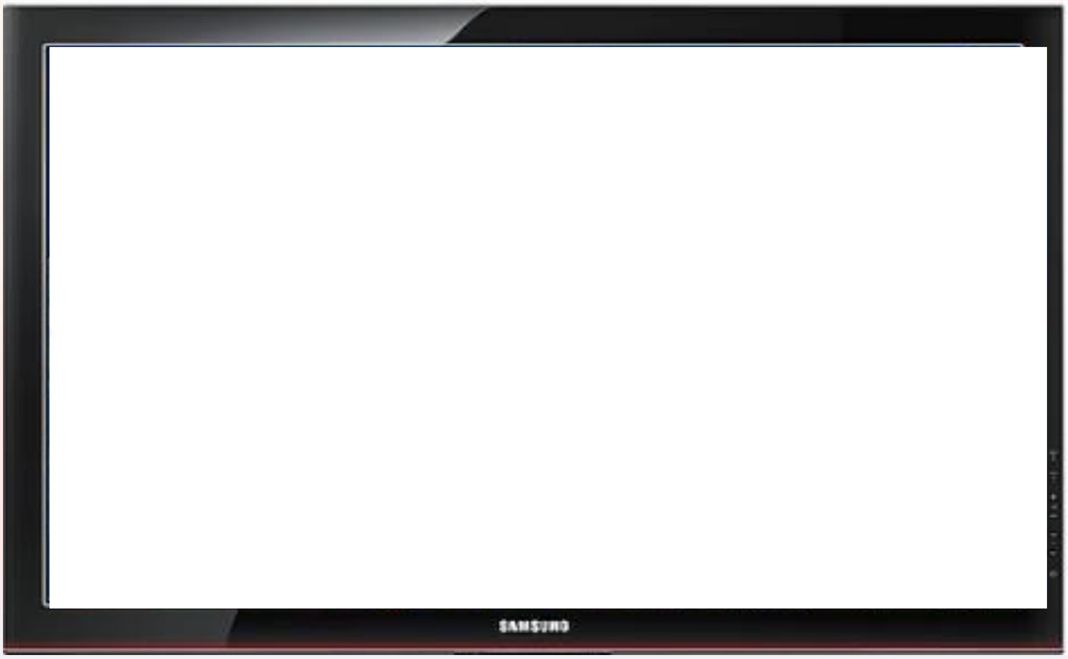




Video
Experiment Impact of Jet

REFRESHMENT

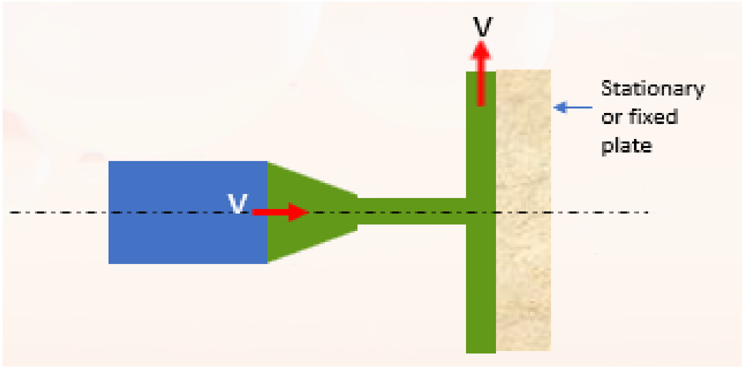
Video
Application of Water Jet



REFRESHMENT

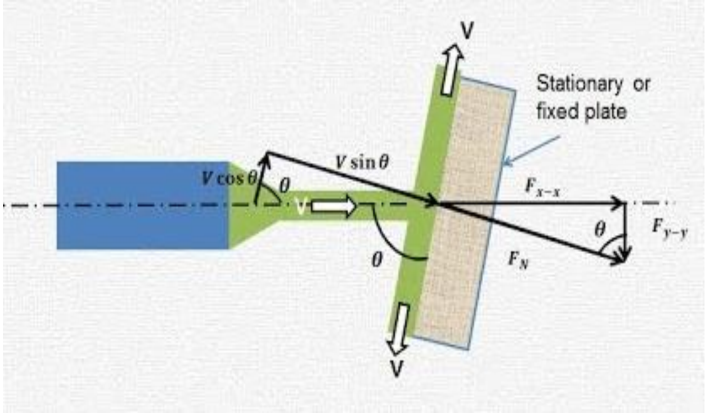
REFRESHMENT

Impact of jet on a stationary flat plate



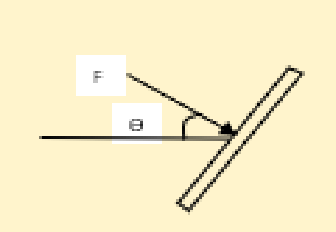
$F = \rho A v^2$

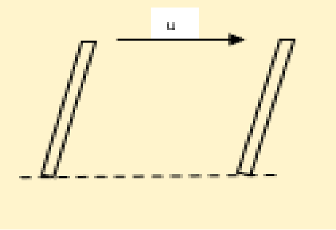
Impact Of A Jet On A Stationary Flat Plate Inclined at an angle Ɵ

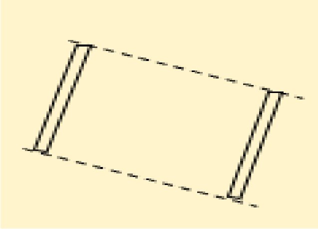


$F = \rho A (v - u)^2 \cos \Theta$

Inclined Plate







Fixed
$F = \rho A v^2 \cos\boldsymbol{\Theta}$

Moving in the direction of the Jet
$F = \rho A (v - u)^2 \cos\boldsymbol{\Theta}$

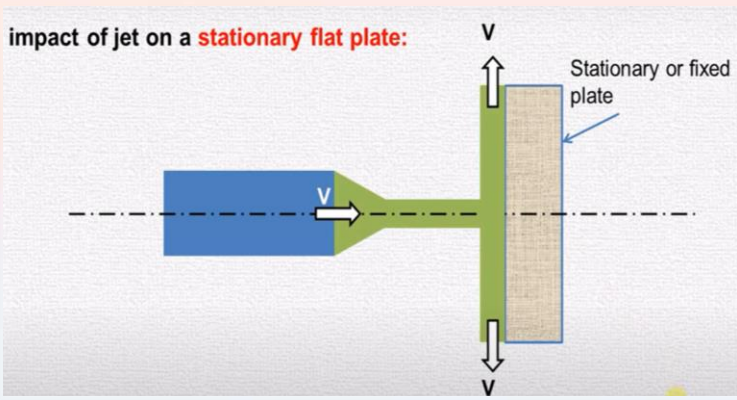
Moving parallel with the plate
$F = \rho A \dfrac{(v-u)}{\cos\theta}(v\cos\theta - u)$

Example 1.2.1

Question 1.2.1

A jet of water 100 mm in diameter hits a fixed flat plate normally. Calculate the force exerted by the jet when its velocity is 30 m/s.

Solution 1.2.1



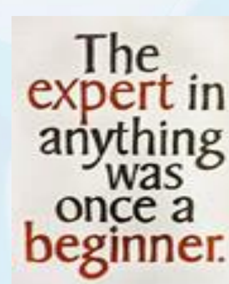
Given;

$$d = 100mm = 0.1\text{m} \quad , \quad V = 30 \text{ m/s}$$

$$A = \frac{\pi(0.1)^2}{4} = 7.854 \times 10^{-3}\text{m}^2$$

Force on the plate (stationary)

$$\begin{aligned} F &= \rho A V^2 \\ &= 1000 \times (7.854 \times 10^{-3}) \times 30^2 \\ &= 7068.6N \\ &= 7.069kN \end{aligned}$$



Example 1.2.2

Question 1.2.2

A jet of water 5cm diameter moves at a rate 36 km/hour hits a fixed flat plate normally. Calculate the force due to impact of jet.

Solution 1.2.2

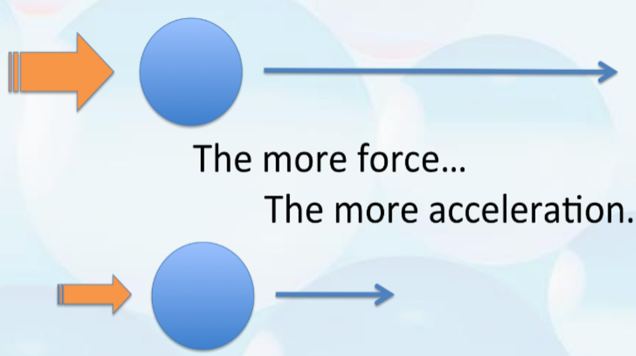
$$d = 5cm = 0.05m$$

$$V = 36\frac{km}{hr} \times \frac{1hr}{3600s} \times \frac{1000m}{1km} = 10m/s$$

$$\mathrm{F} = \rho \mathrm{AV}^2$$

$$= 1000 \times \left(\frac{\pi(0.05)^2}{4}\right) \times 10^2$$

$$= \mathbf{196.35N}$$



Example 1.2.3

Question 1.2.3

A jet of water coming out of a 50 mm diameter nozzle hits a stationary plate at right angles. Determine the impact force of the water jet on the plate if the jet has a velocity of 6.3 m/s.

Solution 1.2.3

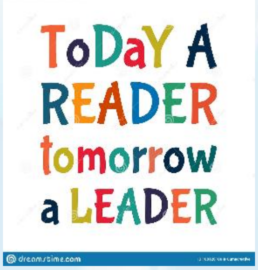
Given;

d = 50mm = 0.05m , V = 6.3 m/s

$$A = \frac{\pi(0.05)^2}{4} = 1.963 \times 10^{-3}\mathrm{m}^2$$

Force on the plate (stationary)

$$\begin{aligned} F &= \rho A V^2 \\ &= 1000 \times (1.963 \times 10^{-3}) \times 6.3^2 \\ &= 77.911N \end{aligned}$$



Example 1.3.1

Question 1.3.1

A jet of oil 15cm diameter (specific gravity = 0.95) hits a fixed flat plate. If the oil jet travels with 16m/s. Calculate the force exerted by the oil on the plate at the angle of 55° to the plate.

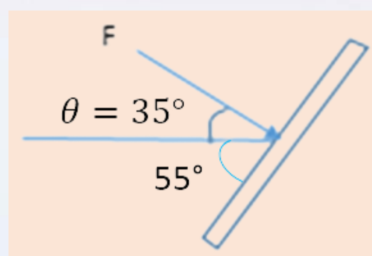
Solution 1.3.1

Given:

$$d = 15\text{cm} = 0.15\text{m}$$

$$v = 16\text{m/s}$$

$$u = 1.67\text{m/s}$$



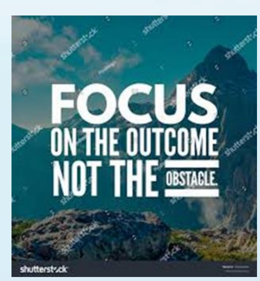
$$0.95 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.95 \times 1000$$
$$= 950 kg/m^3$$

$$F = \rho A v^2 \cos \boldsymbol{\theta}$$

$$= 1000 \times \left(\frac{\pi (0.15)^2}{4}\right) \times 16^2 \times \cos 35$$

$$= \mathbf{3705.757 N}$$



Example 1.3.2

Question 1.3.2

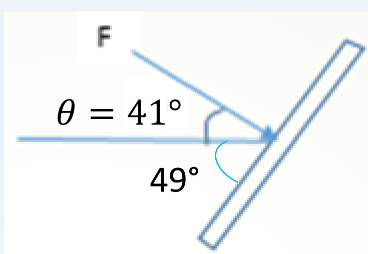
A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18 m/s. Calculate the force on the plate. Calculate the force exerted by the oil on the plate at the angle of 49° to the plate.

Solution 1.3.2

Given;

d = 50mm = 0.5m , V = 18m/s

$$A = \frac{\pi(0.05)^2}{4} = 1.963 \times 10^{-3}\mathrm{m}^2$$



$$F = \rho A v^2 \cos 35°$$

$$= 1000 \times (1.963x10^{-3}) \times (18)^2 \cos 41°$$

$$= \mathbf{480.004N}$$

Example 1.4.1

Question 1.4.1

A jet of water 25 mm diameter moves at a rate of 6 m/s and hits a flat plate capable of moving at a rate 1.5 m/s in the same direction as the jet. Compute the force exerted on the plate if the jet hits the plate normally.

Solution 1.4.1

Given:

$$\mathrm{d} = 25\mathrm{mm} = 0.025\mathrm{m}$$
$$\mathrm{V} = 6\mathrm{m/s}$$
$$\mu = 1.5\mathrm{m/s}$$

$$F = \rho A(v - u)^2$$
$$= 1000 \times \left(\frac{\pi(0.025)^2}{4}\right) \times (6 - 1.5)^2 = \mathbf{9.94}\boldsymbol{N}$$

Example 1.4.2

Question 1.4.2

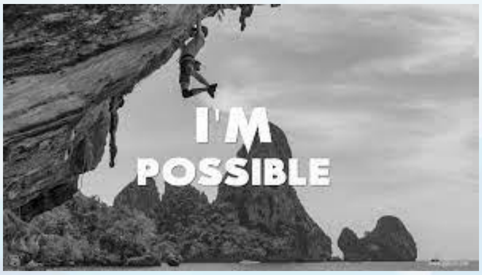
A jet of water 5 cm diameter hits a flat plate at a speed of 33 m/s. If the plate moves at a rate 9 m/s in the same direction as the jet, calculate the force exerted by the jet on the moving plate.

Solution 1.4.2

Given:

$$d = 5\text{cm} = 0.05\text{m}$$
$$V = 33\text{m/s}$$
$$\mu = 9\text{m/s}$$

$$
\begin{aligned}
F &= \rho A(v-u)^2 \\
&= 1000 \times \left(\frac{\pi(0.05)^2}{4}\right) \times (33-9)^2 \\
&= 1130.973N \\
&= \mathbf{1.131kN}
\end{aligned}
$$



Example 1.4.3

Question 1.4.3

A water jet 35mm diameter move at a rate of 6.5m/s and hits a flat plate capable of moving at a rate 1.73m/s in the same direction as the jet. Compute the force exerted on the plate if the angle between the jet and the plate is 60°.

Solution 1.4.3

Given;

$d = 35mm = 0.35\text{m}$

$\text{v} = 6.5\text{m/s}$

$\text{u} = 1.73\text{m/s}$

$$\text{A} = \frac{\pi(0.35)^2}{4} = 96.211 \times 10^{-3}\text{m}^2$$

Moving in the direction of the jet

$$\text{F} = \rho\text{A}(\text{v} - \text{u})^2\cos\boldsymbol{\theta}$$

$$= 1000 \times (96.211x10^{-3}) \times (6.5 - 1.73)^2 \cos 30°$$

$$= 1895.798N$$

QUESTION 1

A water jet 20cm diameter move at a rate of 10.5m/s and hits a flat fixed plate normally. Compute the force exerted on the plate.

QUESTION 2

An 80mm diameter jet has a velocity of 41m/s strikes a flat plate. Calculate the normal pressure on the plate if:

i. The plate is statics

ii. The plate is moving with a velocity of 22m/s and away from the jet.

QUESTION 3

A water jet 10cm diameter strikes a flat plate with the velocity of 20m/s. Calculate the force on the plate if:

i. The plate is statics

ii. The plate is moving with a velocity of 9.5 m/s away from the jet

iii. The plate is inclined the angle between the jet and the plate is 64°

SOLUTION EXERCISE CHAPTER 1

SOLUTION 1

$$\text{Diameter}, d = 20cm = 0.20m$$

$$Velocity, V = 10.5m/s$$

$$Area, A = \frac{\pi d^2}{4} = \frac{\pi(0.2)^2}{4} = 0.031m^2$$

$$\begin{aligned} \text{Force, F} &= \rho A V^2 \\ &= 1000 \times 0.031 \times 10.5^2 \\ &= \mathbf{3417.75N} \end{aligned}$$

SOLUTION
EXERCISE CHAPTER 1

SOLUTION 2

Diameter, $d = 80mm = 0.08m$

$Velocity, V = 41m/s$

$$Area, A = \frac{\pi d^2}{4} = \frac{\pi(0.08)^2}{4} = 5.027 \times 10^{-3} m^2$$

i) Force, $\mathrm{F} = \rho \mathrm{AV}^2$

$$= 1000 \times (5.027 \times 10^{-3}) \times 41^2$$

$$= \mathbf{8450.387}\boldsymbol{N}$$

ii) Force, $\mathrm{F} = \rho \mathrm{A}(\mathrm{V} - U)^2$

$$= 1000 \times (5.027 \times 10^{-3}) \times (41 - 22)^2$$

$$= \mathbf{1814.747}\boldsymbol{N}$$

SOLUTION
EXERCISE CHAPTER 1

SOLUTION 3

$\text{Diameter}, d = 10mm = 0.01m$

$Velocity, V = 20m/s$

$$Area, A = \frac{\pi d^2}{4} = \frac{\pi (0.01)^2}{4} = 7.854 \times 10^{-5} m^2$$

i) $\text{Force}, \text{F} = \rho \text{AV}^2$

$= 1000 \times (7.854 \times 10^{-5}) \times 20^2$

$= \mathbf{31.416N}$

ii) $\text{Force}, \text{F} = \rho \text{A}(\text{V} - U)^2$

$= 1000 \times (5.027 \times 10^{-3}) \times (20 - 9.5)^2$

$= \mathbf{554.227N}$

iii) $\text{Force}, \text{F} = \rho \text{A}(\text{V} - U)^2 \cos \boldsymbol{\theta}$

$= 1000 \times (5.027 \times 10^{-3}) \times (20 - 9.5)^2 \cos 32$

$= \mathbf{470.011N}$

Practice Problems
- Flat Plate-

1. Describe the following law of motion

   i. Newton's second law.

   ii. Newton's third law.

2. A water jet 100 mm in diameter hits a fixed flat plat normally. Calculate the force exerted by the jet when its velocity is 30m/s.

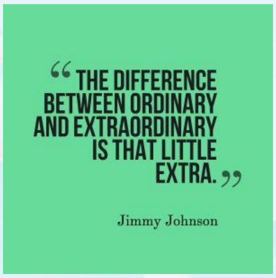
3. A water jet 5 cm hits a flat plate at a speed of 33 m/s. If the plate moves at a rate 9m/s in the same direction as the jet, calculate the force exerted by the jet on the moving plate.

4. A 95 mm diameter jet having a velocity of 50 meters per second strikes a flat plate. Calculate the normal force on the plate.

5. A 75mm jet of an oil having specific gravity 0.8 strikes normally a stationary flat plate. If the force exerted by the jet on the plate is 1200N,calculate the velocity of jet oil. (8m)

6. A water jet 35mm diameter move at a rate of 4.5m/s and hits a flat plate capable of moving at a rate 1.7m/s in the same direction as the jet. Compute the force exerted on the plate if:
   i. The jet hit the plate normally
   ii. The angle between the jet and plate is 60^0.

7. An 85 mm diameter jet has a velocity of 40meters per second strikes a flat plate. Calculate the normal pressure on the plate if:
   i. The plate is static
   ii. The plate is moving with a velocity of 25m/s and away from the jet.



SUMMARY OF THE TOPIC

Principal of Momentum = mv

F = MA (Newton's Second Law)

Momentum in fluid

$$F = m(V_2 - V_1)$$

$$F = \rho Q(V_2 - V_1)$$

FORMULA USED

i. Stationary or fixed plate

$F = \rho Q v = \rho A\, v^2$

ii. Moving plate

$F = \rho A(\ v - u)^2$

iii. Inclined plate

$F = \rho A\, v^2 \cos \Theta$,

Θ= tangential to water jet

CHAPTER 2

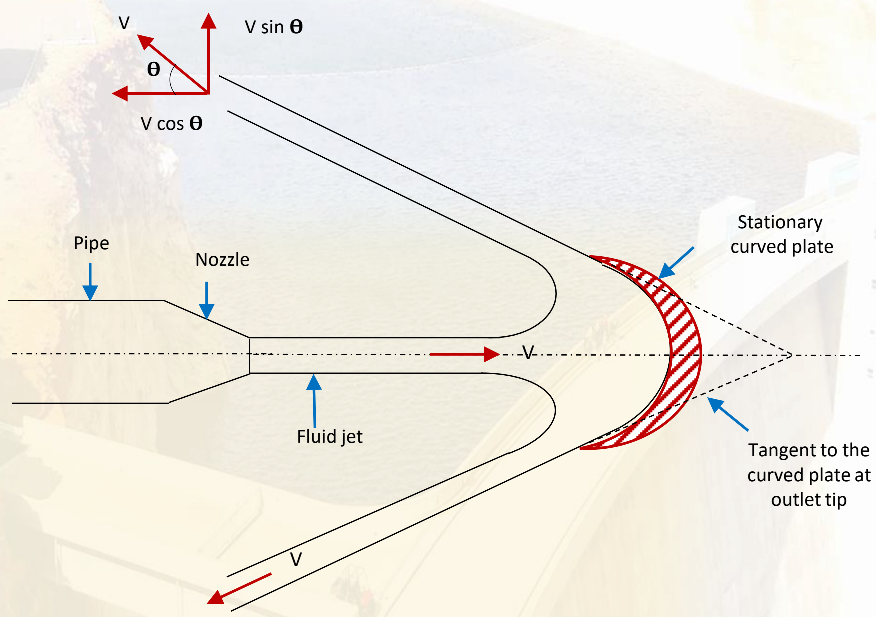
MOMENTUM EQUATION-PROBLEM SOLVING

Impact of Jet on Curved Vane

Force of Impact of Jet on a Fixed Curved Vane

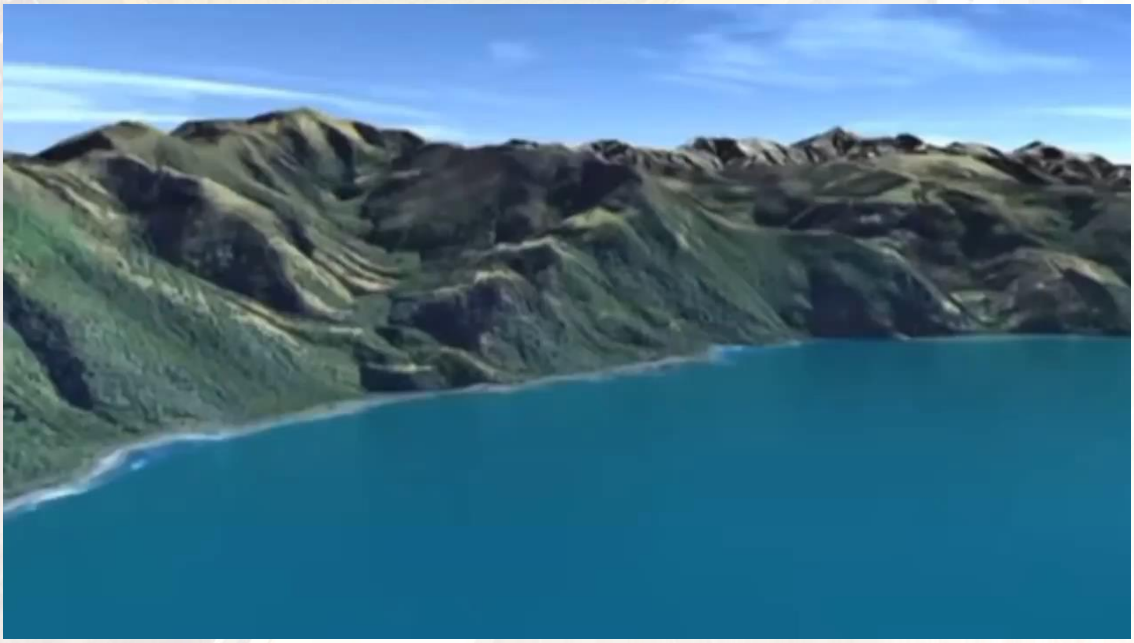
Force on the curve vane is caused by the change of momentum of the liquid

F= rate of flow of mass x change in velocity



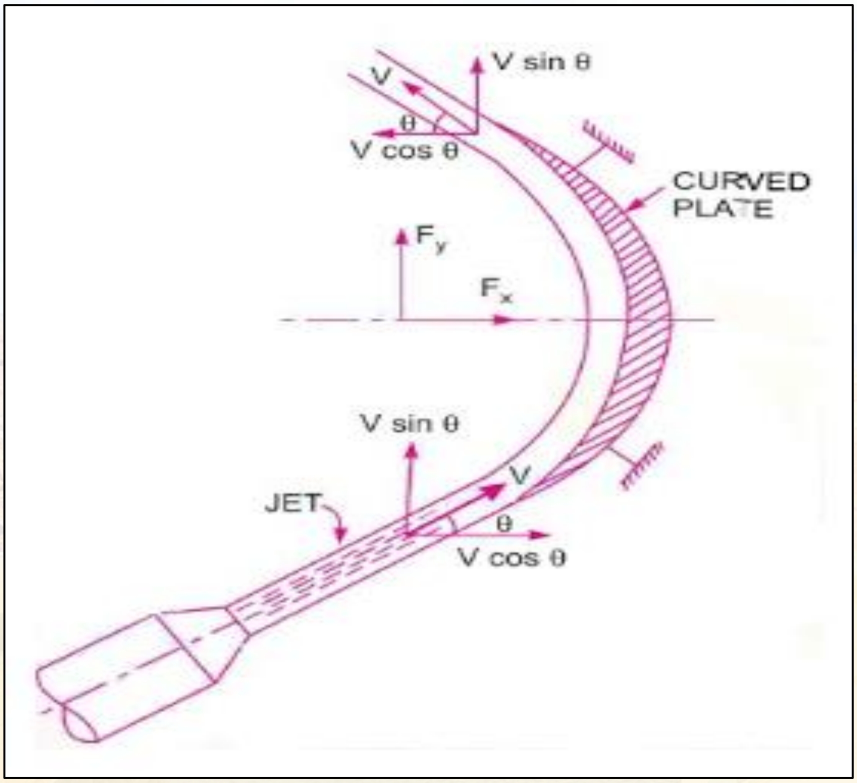
REFRESHMENT

Video
Application of Water Jet on Curve Vane



REFRESHMENT

Force Due To Deflection Of A Jet By Curved Vane



From the equations of momentum, the force components that acts on water jet are;

❖ *Force in X direction*

$$F_x = \rho Q(v_{2x} - v_{1x})$$

❖ *Force in Y direction*

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$\therefore Resultant\ force, F = \sqrt{F_x^2 + F_y^2}$$

$$\text{Direction of resultant force against the x} - \text{axis,}$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x}$$

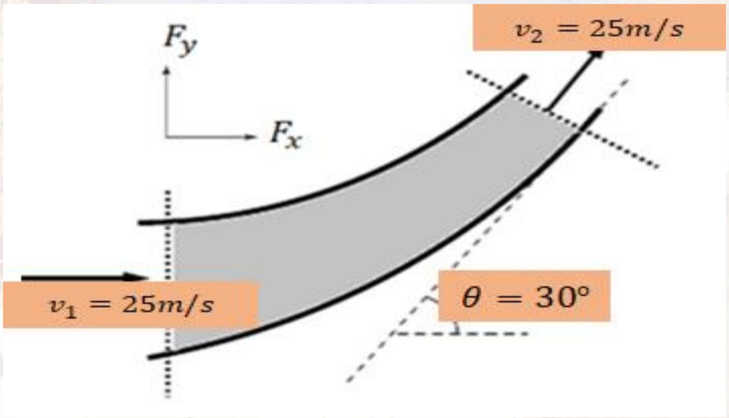
REFRESHMENT

Example 2.2.1

Question 2.2.1

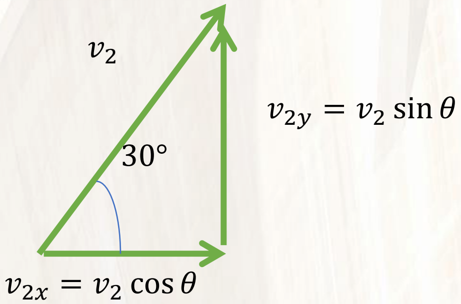
A 10 cm diameter jet of water strikes a curved stationary vane with a velocity of 25 m/s. The angle at inlet is zero and the angle at outlet is 30^0. Calculate the magnitude and direction of the resultant force on the curved vane.

Solution 2.2.1



Water discharge;

$$Q = A \times v$$
$$= \frac{\pi(0.1)^2}{4} \times 25$$
$$= 0.196 m^3/s$$



Solution 2.2.1 cont..

Example 2.2.1 cont..

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$

$$Fx = 1000\,(0.196)(25\cos 30° - 25)$$

$$= -656.476N$$

$$\therefore F_x = 656.476N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$Fy = 1000\,(0.196)(25\sin 30° - 0)$$

$$\therefore F_y = 2450N(\uparrow)$$

$$\text{FR}= \sqrt{F_x^2 + F_y^2}$$

$$\text{FR} = \sqrt{656.476^2 + 2450^2}$$

$$= \mathbf{2.536kN}$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x}$$

$$\emptyset \ = tan^{-1}\frac{2450}{656.476} = \mathbf{75°}$$

$\therefore$ **The resultant force on the curved vane,**
F = 2.536kN,

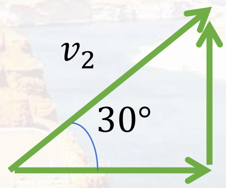
$\therefore$ **Direction of resultant force against the**
x − axis = 75°

Example 2.2.2

Question 2.2.2

A water jet from 50mm diameter nozzle is deflected through 60° at 36m/s above a fountain site. The velocity of water leaves from the curved vane is 30m/s due to friction. Calculate the magnitude and direction of the resultant force on the curved vane.

Solution 2.2.2



$$v_{2y} = v_2 \sin\theta$$

$$v_{2x} = v_2 \cos\theta$$

$$Q = A \times v$$
$$= \frac{\pi(0.1)^2}{4} \times 25$$
$$= 0.196 m^3/s$$

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$F_x = 1000\,(0.196)(25\cos 30° - 25)$$
$$F_x = -656.476N$$
$$\therefore F_x = 656.476N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$
$$F_y = 1000\,(0.196)(25\sin 30° - 0)$$
$$F_y = 2450N$$
$$\therefore F_y = 2450N(\uparrow)$$

$$\mathrm{Fr} = \sqrt{F_x^2 + F_y^2}$$
$$F_y = \sqrt{656.476^2 + 2450^2}$$
$$= \mathbf{2.536}\boldsymbol{kN}$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{2450}{656.476} = \mathbf{75°}$$

$\therefore$ **The resultant force on the curved vane, F = 2.536*kN*,**
and direction of resultant force against the x − axis = 75°

Example 2.2.3

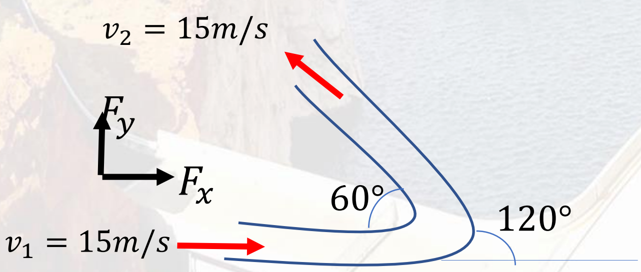
Question 2.2.3

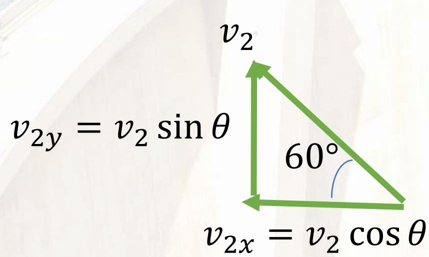
A jet of water flows tangentially onto a single stationary vane as shown in the figure below, with an initial velocity,v1 of 16 m/s. The jet is turned through 1200 by the vane ad has an exit velocity of v2. The flow rate of the jet is 0.04 m3/s. Calculate the magnitude and direction of the resultant force exerted on the curved vane if:

i. The vane assumed to be smooth

ii. The exit velocity is 85% of the initial flow velocity

Solution 2.2.3





$$Q = 0.04m^3/s$$

$$v_{1x} = 16m/s$$

$$v_{1y} = 0m/s$$

$$\sin 60° = \frac{v_{2y}}{v_2}$$
$$v_{2y} = 16\, sin60$$
$$= 13.856m/s$$

$$\cos\theta = \frac{v_{2x}}{v_2}$$
$$v_{2y} = 16\, cos60°$$
$$= -8m/s$$

Solution 2.2.3 cont..

a) **Vane is assumed to be smooth, $\mathbf{v_1 = v_2}$**

$$\underline{\boldsymbol{Force\ in\ X\ direction}}$$

$$F_x = \rho Q(v_{2x} - v_{1x})$$

$$F_x = 1000\ (0.04)(-8 - 16)$$

$$F_x\ = -960N$$

$$\therefore F_x = 960\ N(\leftarrow)$$

$$\underline{\boldsymbol{Force\ in\ Y\ direction}}$$

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$F_y = 1000\ (0.04)(13.856 - 0)$$

$$F_y\ = 554.24\ N$$

$$\therefore F_y = 554.24\ N(\uparrow)$$

$$\mathrm{Fr} = \sqrt{F_x^2 + F_y^2}$$

$$F_y\ = \sqrt{(960)^2 + (554.24)^2}$$

$$\boldsymbol{= 1108.504N}$$

$$\boldsymbol{= 1.109kN}$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{554.24}{960} \boldsymbol{= 30°}$$

$\therefore$ **The resultant force on the curved vane, $\mathbf{F = 1.109}\boldsymbol{kN}$,**
and direction of resultant force against the x − axis = 30°

ution 2.2.3 cont..

Example 2.2.3 cont..

b) The exit velocity is 85% of the initial flow velocity

$$\text{Exit velocity}, v_2 = 85\% v_1$$

$$= \frac{85}{100} \times 16 = 13.6 m/s$$

$$\sin 60° = \frac{v_{2y}}{v_2}$$

$$v_{2y} = 13.6 \sin 60$$

$$= 11.778 m/s$$

$$\cos\theta = \frac{v_{2x}}{v_2}$$

$$v_{2y} = 13.6 \cos 60°$$

$$= -6.8 m/s$$

$$v_{1x} = 16 m/s$$

<u>Force in X direction</u>

$$F_x = \rho Q(v_{2x} - v_{1x})$$

$$F_x = 1000\,(0.04)(-6.8 - 16)$$

$$F_x = -912 N$$

$$\therefore F_x = 912\, N(\leftarrow)$$

<u>Force in Y direction</u>

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$F_y = 1000\,(0.04)(11.778 - 0)$$

$$F_y = 471.12\, N$$

$$\therefore F_y = 471.12\, N(\uparrow)$$

$$\text{Fr} = \sqrt{F_x^2 + F_y^2}$$

$$F_y = \sqrt{(912)^2 + (471.12)^2}$$

$$= \mathbf{1026.48 N}$$

$$= \mathbf{1.026 kN}$$

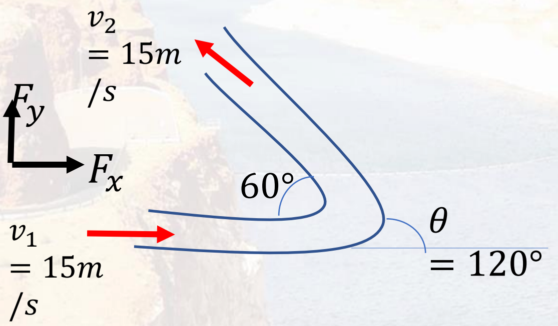
$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{471.12}{912} = \mathbf{27.3°}$$

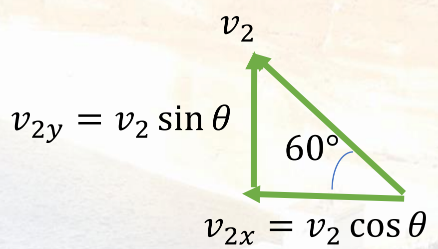
$\therefore$ **The resultant force on the curved vane, F = 1.026kN,**
and direction of resultant force against the x − axis = 27.3°

Question 2.2.4

A water jet through 6 mm diameter and with a velocity of 15 m/s strikes a curved vane. The jet deflected through 120° by the vane. Calculate the magnitude and direction of the resultant force.

Solution 2.2.4





$$Q = A \times v = \frac{\pi(0.006)^2}{4} \times 15 = 4.241 \times 10^{-4} m^3/s$$

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$= 1000\,(4.241 \times 10^{-4})(-15\cos 60° - 15) = -9.542N$$
$$\therefore F_x = 9.542N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q(v_{2y} - v_{1y})$$
$$= 1000\,(4.241 \times 10^{-4})(15\sin 60° - 0) = 5.509N$$
$$\therefore F_y = 5.509N(\uparrow)$$
$$F = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{9.542^2 + 5.509^2} = 11.018N$$
$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{5.509}{9.542} = 30°$$

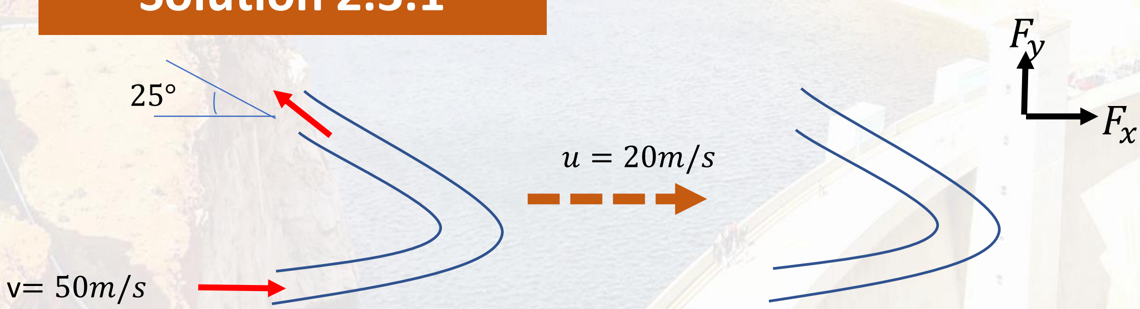
$\therefore$ **The resultant force on the curved vane, F = 11.018*kN*,**
and direction of resultant force against the x − axis = 30°

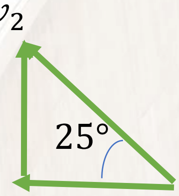
Question 2.3.1

curved surface (vane) having an inlet angle of zero degrees and an outlet angle of 25º receives a jet of water at velocity of 50m/s. If the vane is moving with a velocity of 20m/s in the direction of the jet, calculate the force components in direction in direction of the vane velocity and across it. Then determine the magnitude and direction of the resultant force acting on the vane. Discharge 1000liter/s.

Example 2.3.1

Solution 2.3.1





$$v_{2y} = v_2 \sin\theta$$

$$v_{2x} = v_2 \cos\theta$$

$$Q = \frac{1000litre}{s} = 1m^3/s$$

$$effective\ velocity = v - u = 50 - 20 = 30m/s$$

$$v_{1x} = 30m/s$$

$$v_{1y} = 0m/s$$

$$\sin 25° = \frac{v_{2y}}{v_x}$$

$$v_{2y} = 30\ sin25 = 12.68m/s$$

$$\cos\theta = \frac{v_{2x}}{v_x}$$

$$v_{2y} = 30\ cos25° = 27.19m/sm/s$$

Solution 2.3.1 cont..

Example 2.3.1 cont..

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$= 1000\,(1)(-27.19 - 30) = -57190N$$
$$\therefore F_x = 57190\,N(\leftarrow)$$

Force in Y direction

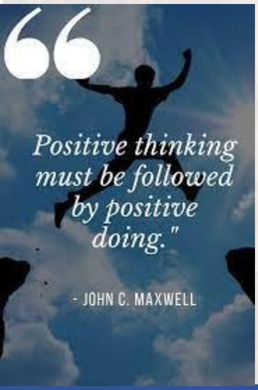
$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$
$$= 1000\,(1)(12.68 - 0) = 12680N$$
$$\therefore F_y = 12680N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{(-57190)^2 + 12680^2} = 58578.823N$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{12680}{57190} = 12.50°$$

$\therefore$ **The resultant force on the curved vane, F = 58.579kN,**

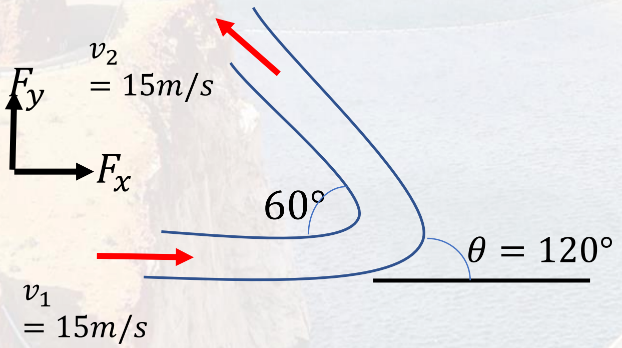
and direction of resultant force against the x − axis = 12.50°



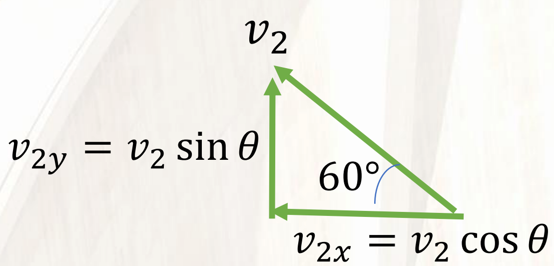
Question 2.3.2

A A jet of water 6 mm diameter and moving with a velocity of 15 m/s strikes a curved vane. The jet deflected through 1200 by the vane. Calculate the magnitude and direction of the resultant force.

Solution 2.3.2



$$Q = A \times v$$
$$= \frac{\pi(0.006)^2}{4} \times 15$$
$$= 4.241 \times 10^{-4} m^3/s$$



$$\sin 25° = \frac{v_{2y}}{v_x}$$
$$v_{2y} = 30\, sin25$$
$$= 12.68m/s$$

$$\cos\theta = \frac{v_{2x}}{v_x}$$
$$v_{2y} = 30\, cos25°$$
$$= 27.19m/sm/s$$

Solution 2.3.2 cont..

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$

$$= 1000\,(4.241 \times 10^{-4})(-15\cos 60° - 15) = -9.542N$$

$$\therefore F_x = 9.542N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$= 1000\,(4.241 \times 10^{-4})(15\sin 60° - 0) = 5.509N$$

$$\therefore F_y = 5.509N(\uparrow)$$

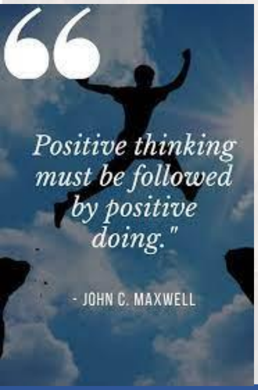
$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{9.542^2 + 5.509^2} = 11.018N$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{5.509}{9.542} = 30°$$

$\therefore$ **The resultant force on the curved vane, F = 11.018*kN*,**

and direction of resultant force against the x − axis = 30°



EXERCISE CHAPTER 2

QUESTION 1

A water jet with a diameter of 50mm is deflected by 50° at the velocity of the blade . Calculate the magnitude of the force generated by water on the blade when the velocity of water jet leave the blade is 35m/s due to friction

QUESTION 2

A jet of water 50mm diameter and having a velocity of 25 m/s enters tangentially a stationary curved vane and deflected through an angle of 45^0. Calculate the magnitude and direction of the resultant force on the vane.

QUESTION 3

A jet of water from a nozzle is deflected through 60^0 from its original direction by the curved plate which enter it tangentially with a velocity of 30 m/s and leaves with a velocity of 25 m/s. If the discharge from the nozzle is 0.8 m/s, calculate the magnitude and direction of the resultant force on the vane if the vane stationary. (22.27N, 51^0.

SOLUTION
EXERCISE CHAPTER 2

SOLUTION 1

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.05)^2}{4}$$
$$A = 1.963 \times 10^{-3} m^2$$

$$Q = A \times v$$
$$Q = 1.936 \times 10^{-3} \times 35$$
$$Q = 0.068 m^3/ss$$

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$= 1000\,(0.068)(35\cos 50° - 35) = -850.17N$$
$$\therefore F_x = 850.17N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$
$$F_y = 1000\,(0.068)(35\sin 50° - 0) = 1823.19N$$
$$\therefore F_y = 1823.19N(\uparrow)$$
$$FR = \sqrt{F_x^2 + F_y^2}$$
$$FR = \sqrt{(-850.17)^2 + (1823.19)^2} = 2011.59N$$
$$\emptyset = tan^{-1}\frac{1823.19}{-850.17} = -65°$$

$\therefore$ The resultant force on the curved vane,

F = 2011.59N, and direction of resultant force against the x – axis = 65°

SOLUTION EXERCISE CHAPTER 2

SOLUTION 2

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.05)^2}{4}$$
$$A = 1.963 \times 10^{-3} m^2$$

$$Q = A \times v$$
$$Q = 1.936 \times 10^{-3} \times 25$$
$$Q = 0.0484 m^3/ss$$

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$= 1000\,(0.0484)(25\cos 45° - 25) = -354.40N$$
$$\therefore F_x = 354.40N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$
$$F_y = 1000\,(0.0484)(25\sin 45° - 0) = 855.59N$$
$$\therefore F_y = 855.59N(\uparrow)$$
$$FR = \sqrt{F_x^2 + F_y^2}$$
$$FR = \sqrt{(-354.40)^2 + (855.59)^2} = 926.09N$$
$$\emptyset = tan^{-1}\frac{855.59}{-354.40} = -67.5°$$

$\therefore$ The resultant force on the curved vane,

F = 926.09N,

and direction of resultant force against the x – axis = 67.5°

SOLUTION
EXERCISE CHAPTER 2

SOLUTION 3

Given; $Q = 0.8m^3/s$

Force in X direction

$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$F_x = 1000\,(0.8)(25\cos 60° - 30) = -14000N$$
$$\therefore F_x = 14000N(\leftarrow)$$

Force in Y direction

$$F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$
$$F_y = 1000\,(0.8)(25\sin 60° - 0) = 17320.51N$$
$$\therefore F_y = 17320.51N(\uparrow)$$
$$FR = \sqrt{F_x^2 + F_y^2}$$
$$FR = \sqrt{(-14000)^2 + (17320.51)^2} = 22271.06N$$
$$\emptyset = tan^{-1}\frac{17320.51}{-14000} = -51.1°$$

$\therefore$ The resultant force on the curved vane,

FR = 22271.06N,

and direction of resultant force against the x − axis, $\emptyset = 51.5°$

Practice Problems - Curve Vane -

1. Express Newton's first, second and third laws.

2. A jet of water strikes a single vane, which reverses it through 180° without friction loss. If the jet has an area of $2500mm^2$ and velocity of 55m/s, find the forced exerted if the vane moves :
i. in the same direction as the jet with a velocity of 20m/s
ii. in a direction opposite to that of the jet with a velocity 20m/s.

(Ans: (a) 6.13 kN, (b) = 28.1 kN)

3. A free jet of water with an initial diameter of 2.54cm strikes the vane shown in Figure 1. Given that θ = 30° and V_1 = 30.48m/s. Owing to friction losses assume that V_2 = 28.96m/s. Flow occurs in a horizontal plane. Calculate the resultant force on the blade.

(Ans: F_x = 333.616 N←, F_y = 894.092 N)

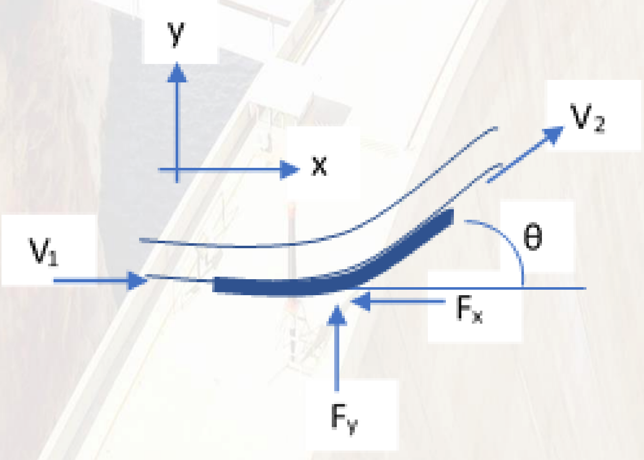


Figure 1

Practice Problems
- Curve Vane -

4. A 100mm diameter water jet with a velocity of 35m/s acts on a series of vanes with $\alpha 1 = \beta 1 = 0$. Neglect friction and find the required blade angle $\beta 2$ in order that the resultant force acting on the vane in the direction of the jet is 950N. Solve using vane velocities of 0, 5, 15 and 25 m/s. Also find the maximum possible vane velocity.

(Ans: 25.7°, 27.8°, 34.2°, 49.1°, 33.3 m/s)

5. A jet of water exits from a 25 mm diameter nozzle and hits a stationary curved blade as shown in Figure 2. Calculate the x and y components used to hold the curve blade.

(Ans: Fx = 16.27 N←, Fy = 18.05 N↑)

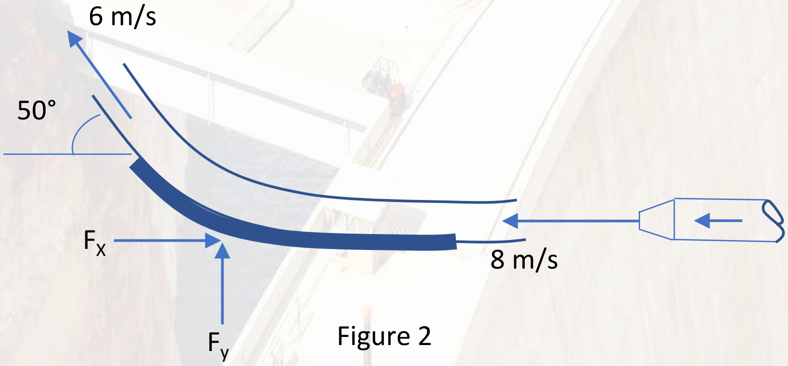


Figure 2

Practice Problems - Curve Vane -

6. A jet of water discharging 40 kg/s exits from a 70 mm diameter nozzle and hits a stationary curved blade as shown in Figure 3. Calculate forces Fx and Fy to hold the blade.

(Ans: Fx = 525.2 N←, Fy = 300.7 N↑)

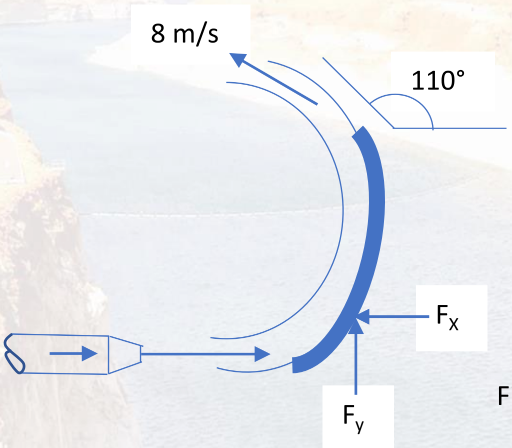


Figure 3

7. Water is flowing out of a nozzle and hits a stationary curved blade as shown in Figure 4. The flow rate in the nozzle is 0.8 kg/s. The flow velocities are v_1 = 7 m/s and v_2 = 6 m/s. Calculate forces, F_x and F_y to hold the blade.

(Ans: F_x = 2.515 N, F_y = 3.677 N)

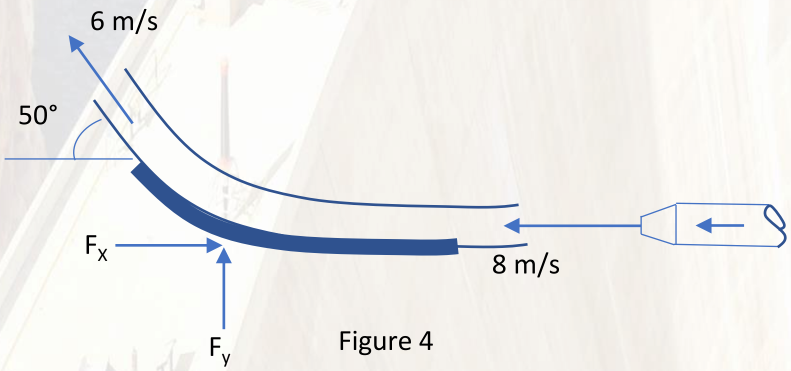


Figure 4

Practice Problems
- Curve Vane -

8. In **Figure 5** assume that friction is negligible, that θ = 115°, and that the water jet has a velocity of 28.96m/s and a diameter of 2.54cm. Calculate:

i. the component of the force acting on the blade in the direction on the jet

ii. the force component normal to the jet

iii. the magnitude of the resultant force exerted on the blade.

Ans: (a) 604.513 N→, (b) = 384.771 N↑, (c) = 716.608 N at 32.5°)

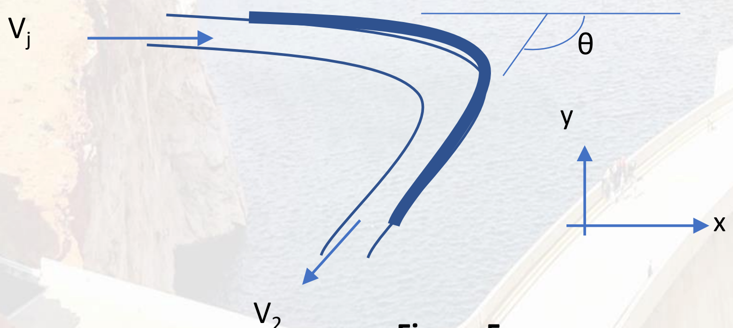


Figure 5

9. By using **Figure 5** assume that friction is negligible, that θ = 115°, and that the water jet has a velocity of 25m/s and a diameter of 40 mm. Calculate:

i. the component of the force acting on the blade in the direction on the jet

ii. the force component normal to the jet

iii. the magnitude of the resultant force exerted on the blade.

(Ans: (a) 1117 N→, (b) = 712 N↑, (c) = 1325 N at 32.5°)

SUMMARY OF THE TOPIC

Principal of Momentum = mv

F = MA (Newton's Second Law)

Momentum in fluid

$$F = m(V_2 - V_1)$$

$$F = \rho Q(V_2 - V_1)$$

FORMULA USED

Force act on y-direction

$$\boldsymbol{\Sigma F_x = \rho Q(v_{2x} - v_{1x})}$$

Force act on y-direction

$$\boldsymbol{\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right)}$$

Resultant force, $\boldsymbol{F = \sqrt{F_x^2 + F_y^2}}$

Direction of resultant,

$$\boldsymbol{\emptyset = tan^{-1}\frac{F_y}{F_x}}$$

CHAPTER 3

MOMENTUM EQUATION- PROBLEM SOLVING

Impact of a Jet on Pipe Bends

Principe Of Bernoulli's Equation In Pipe Flow

The relationship between pressure and velocity in pipe flow – Bernoulli's Theorem

ΣQ in = ΣQ out

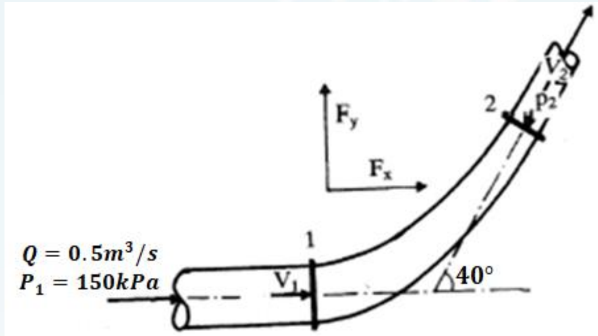
Energy per unit volume before = Energy per unit volume after (by ignored all losses)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

Video of Bernoulli's Equation

REFRESHMENT

Force Due To Deflection Of A Jet in Pipe Bends



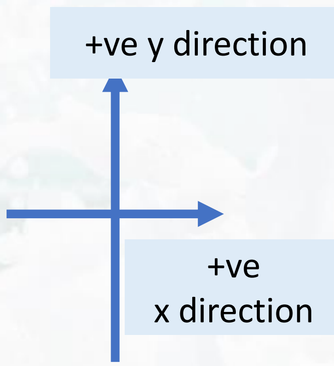
REFRESHMENT

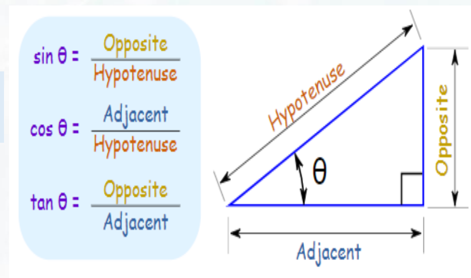
Pressure,

$$P = \frac{F}{A}$$

Force,

$$F = PA$$





From the equations of momentum, the force components that acts on water jet are;

❖ *Force in X direction*

$$\Sigma F_x = \rho Q(v_{2x} - v_{1x})$$

$$P_1A_1 + F_x - P_2A_2 \cos\theta = \rho Q(v_2 \cos\theta - v_1)$$

❖ *Force in Y direction*

$$\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$0 + F_y - P_2A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)$$

$$\therefore Resultant\ force, FR = \sqrt{F_x^2 + F_y^2}$$

Direction of resultant force against the x − axis,

$$\emptyset = tan^{-1}\frac{F_y}{F_x}$$

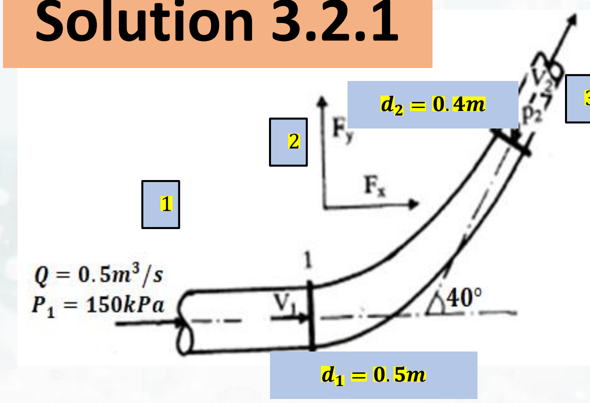
Example 3.2.1

A pipeline bends in the horizontal plane through an angle of 40^0. The diameter of the pipe changes from 60 cm before the bend to 40 cm after it. Water enters the bend at the rate of 500 liters/sec with a pressure of 150 kN/m^2. Calculate

i. the magnitude

ii. direction of the force exerted on the pipe bend

Solution 3.2.1



$$A_1 = \frac{\pi(d_1)^2}{4}$$
$$= \frac{\pi(0.6)^2}{4} = 0.283m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4}$$
$$= \frac{\pi(0.4)^2}{4} = 0.126m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.5}{0.283}$$
$$= 1.767m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.5}{0.126}$$
$$= 3.968m/s$$

$$\boldsymbol{Use\ Bernoulli's\ equation, at\ horizontal\ plane, Z_1 = Z_2}$$

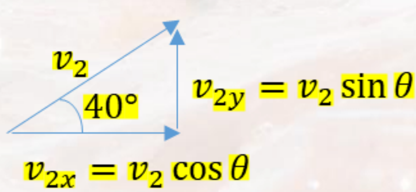
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

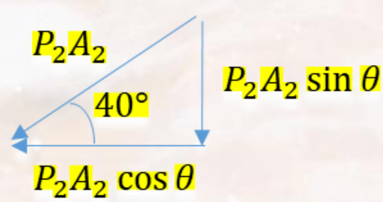
$$\left(\frac{150 \times 10^3}{1000 \times 9.81}\right) + \left(\frac{1.767^2}{2 \times 9.81}\right) = \left(\frac{P_2}{1000 \times 9.81}\right) + \left(\frac{3.968^2}{2 \times 9.81}\right)$$

$$15.45 = \left(\frac{P_2}{9810}\right) + 0.802$$

$$P_2 = 14.648 \times 9810$$

$$P_2 = 143{,}696.88N/m^2$$





<u>***Force in X direction***</u>

$$\Sigma \boldsymbol{F_x} = \boldsymbol{\rho Q}(\boldsymbol{v_{2x}} - \boldsymbol{v_{1x}})$$

$$P_1A_1 + F_x - P_2A_2 \cos\theta = \rho Q(v_2 \cos\theta - v_1)$$

$$150 \times 10^3\ (0.283) + F_x - 143.697 \times 10^3 (0.126) \cos 40° = 1000\ (0.5)(3.968 \cos 40° - 1.767)$$

$$F_x = -27{,}943.803N$$

$$\therefore F_x = 27{,}943.803N(\leftarrow)$$

<u>***Force in Y direction***</u>

$$\Sigma \boldsymbol{F_y} = \boldsymbol{\rho Q}(\boldsymbol{v_{2y}} - \boldsymbol{v_{1y}})$$

$$0 + F_y - P_2A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)$$

$$0 + F_y - 143.697 \times 10^3 (0.126) \sin 40° = 1000(0.5)(3.968 \sin 40° - 0)$$

$$F_y = 12{,}913.489N$$

$$\therefore F_y = 12{,}913.489N(\uparrow)$$

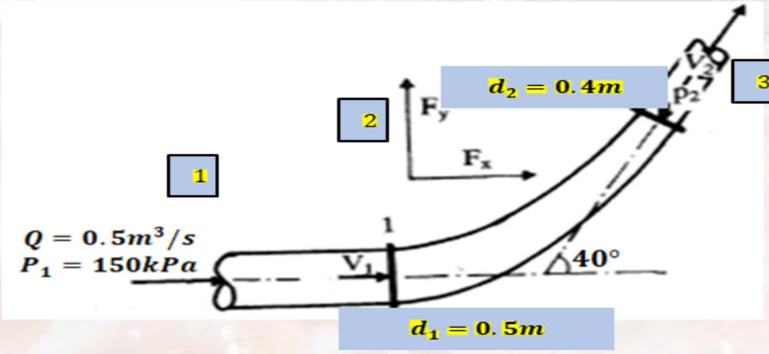
$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{27{,}943.803^2 + 12{,}913.489^2} = 30{,}783.345N$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{12{,}913.489}{27{,}943.803} = 24.8°$$

$\therefore$ **The resultant force on the pipe bend, F = 30, 783. 345N,**

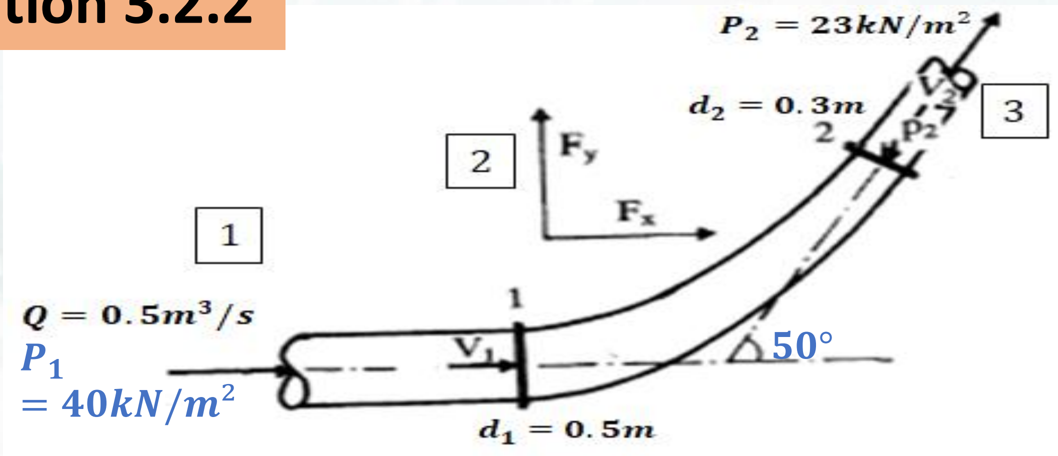
and direction of resultant force against the x − axis = 24. 8°



Example 3.2.2

Water is flowing in a pipe which tapers from diameter d_1 = 500 mm at inlet to d_2 = 300 mm at outlet, and turn through an angle of deflection, θ = 50^0. Pressure at inlet P_1 = 40 kN/m² and at outlet P_2 = 23 kN/m². Calculate the magnitude and direction of the resultant force acting on the bend if flowrate is 500 lit/s.

Solution 3.2.2

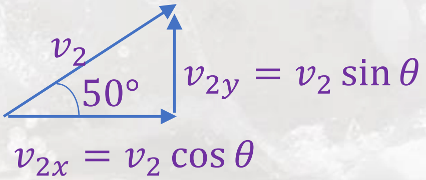


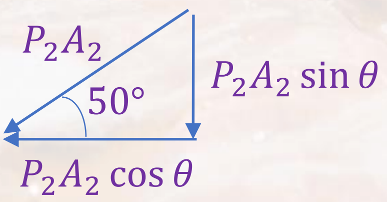
$$A_1 = \frac{\pi(d_1)^2}{4} = \frac{\pi(0.5)^2}{4} = 0.196m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4} = \frac{\pi(0.3)^2}{4} = 0.071m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.5}{0.196} = 2.551m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.5}{0.071} = 7.042m/s$$





$$\underline{\boldsymbol{Force\ in\ X\ direction}}$$

$$\boldsymbol{\Sigma F_x = \rho Q(v_{2x} - v_{1x})}$$

$$\boldsymbol{P_1A_1 + F_x - P_2A_2 \cos\theta = \rho Q(\ v_2 \cos\theta - v_1)}$$

$$40 \times 10^3\ (0.196) + F_x - 23 \times 10^3 (0.071) \cos 50°$$
$$= 1000\ (0.5)(7.042 \cos 50° - 2.551)$$

$$F_x = -5802.573N$$

$$\therefore F_x = 5802.573N(\leftarrow)$$

$$\underline{\boldsymbol{Force\ in\ Y\ direction}}$$

$$\boldsymbol{\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right)}$$

$$\boldsymbol{0 + F_y - P_2A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)}$$

$$0 + F_y - 23 \times 10^3 (0.071) \sin 50° = 1000(0.5)(7.042 \sin 50° - 0)$$

$$F_y = 3948.193N$$

$$\therefore F_y = 3948.193N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{5802.573^2 + 3948.193^2} = 7018.41N$$

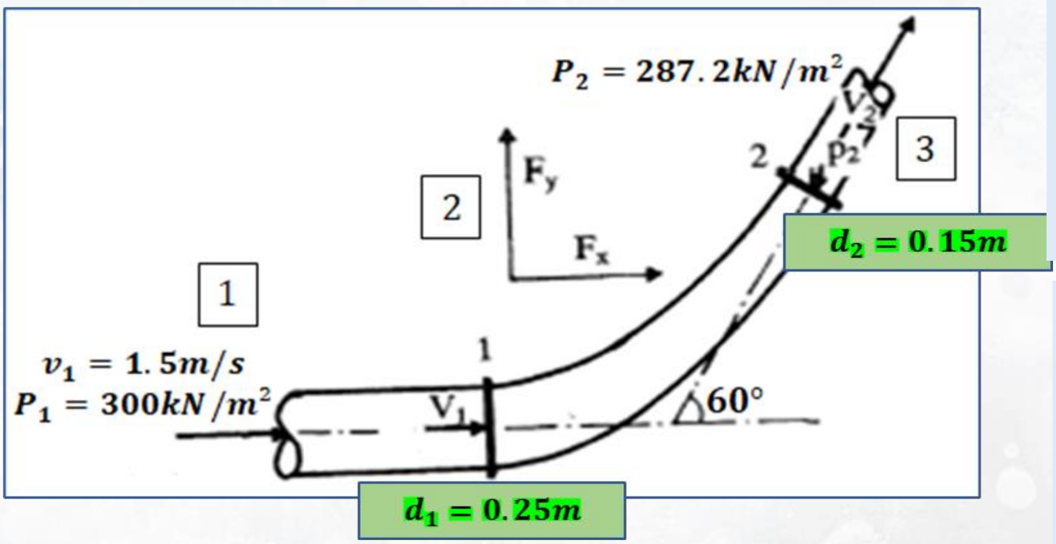
$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{3948.193}{5802.573} = 34.2°$$

$\therefore$ **The resultant force on the pipe bend, F = 7018.41N**
and direction of resultant force against the x − axis = 34.2°

Example 3.2.3

A reducing bend is turned through 60^0 in the horizontal plane and the pipe diameter is reduced from 0.25 m to 0.15m. The velocity and pressure at the entry to the bend are 1.5 m/s and 300 kN/m^2 gauge respectively and at the exit the pressure is 287.2 kN/m^2 gauge. Determine the magnitude and direction of the resultant force on the pipe bend.

Solution 3.2.3



$$A_1 = \frac{\pi(d_1)^2}{4} = \frac{\pi(0.25)^2}{4} = 0.049m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4} = \frac{\pi(0.15)^2}{4} = 0.018m^2$$

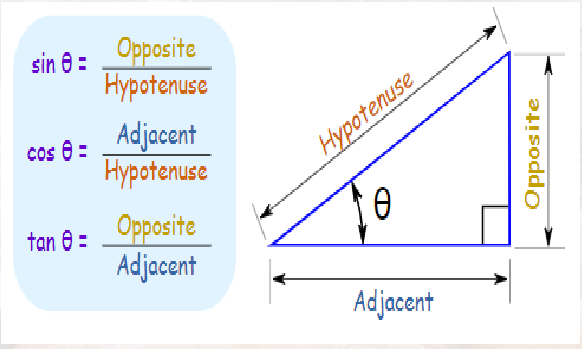
Continuity Equation

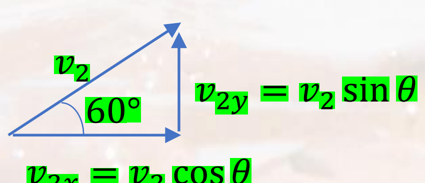
$$Q_1 = Q_2$$

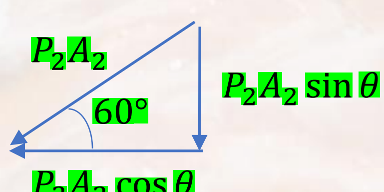
$$A_1 \times v_1 = A_2 \times v_2$$

$$0.049 \times 1.5 = 0.018 \times v_2$$

$$v_2 = \frac{0.074}{0.018} = 4.111m/s$$







Solution 3.2.3 cont.,

$$\underline{\boldsymbol{Force\ in\ X\ direction}}$$

$$\Sigma \boldsymbol{F_x = \rho Q(v_{2x} - v_{1x})}$$

$$\boldsymbol{P_1 A_1 + F_x - P_2 A_2 \cos\theta = \rho Q(\, v_2 \cos\theta - v_1)}$$

$$300 \times 10^3\ (0.049) + F_x - 287.2 \times 10^3 (0.018) \cos 60°$$
$$= 1000\ (0.074)(4.111 \cos 60° - 1.5)$$

$$F_x = -12{,}074.093N$$

$$\therefore F_x = 12{,}074.093N(\leftarrow)$$

$$\underline{\boldsymbol{Force\ in\ Y\ direction}}$$

$$\Sigma \boldsymbol{F_y = \rho Q\left(v_{2y} - v_{1y}\right)}$$

$$\boldsymbol{0 + F_y - P_2 A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)}$$

$$0 + F_y - 287.2$$
$$\times 10^3 (0.018) \sin 60° = 1000(0.074)(4.111 \sin 60° - 0)$$

$$F_y = 4740.462N$$

$$\therefore F_y = 4740.462N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{12{,}074.093^2 + 4740.462^2} = 12{,}971.342N$$

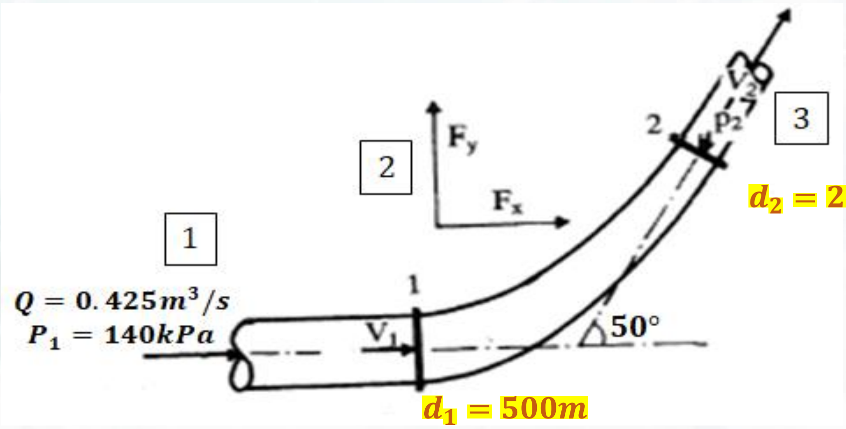
$$\emptyset = tan^{-1} \frac{F_y}{F_x} = tan^{-1} \frac{4740.462}{12{,}074.093} = 21.4°$$

$$\therefore \textbf{The resultant force on the pipe bend, F} = \boldsymbol{12,971.342N}$$
$$\textbf{and direction of resultant force against the x} - \textbf{axis} = \boldsymbol{21.4°}$$

Example 3.2.4

A 500 reducing pipe bend tapers from 500 mm diameter at inlet to 200 mm diameter at outlet. The pressure at inlet is 140 kN/m2 gauge and the flowrate is 0.425 m3/s. Determine the resultant force and direction exerted on the bend.

Solution 3.2.4



$$A_1 = \frac{\pi(d_1)^2}{4}$$
$$= \frac{\pi(0.5)^2}{4} = 0.196m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4}$$
$$= \frac{\pi(0.2)^2}{4} = 0.031m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.425}{0.196}$$
$$= 2.168m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.425}{0.031}$$
$$= 13.71m/s$$

$$\boldsymbol{Use\ Bernoulli's\ equation, at\ horizontal\ plane, Z_1 = Z_2}$$

$$\frac{\boldsymbol{P_1}}{\boldsymbol{\rho g}} + \frac{\boldsymbol{V_1^2}}{\boldsymbol{2g}} + \boldsymbol{Z_1} = \frac{\boldsymbol{P_2}}{\boldsymbol{\rho g}} + \frac{\boldsymbol{V_2^2}}{\boldsymbol{2g}} + \boldsymbol{Z_2}$$

$$\left(\frac{140 \times 10^3}{1000 \times 9.81}\right) + \left(\frac{2.168^2}{2 \times 9.81}\right) = \left(\frac{P_2}{1000 \times 9.81}\right) + \left(\frac{13.71^2}{2 \times 9.81}\right)$$

$$14.511 = \left(\frac{P_2}{9810}\right) + 9.58$$

$$P_2 = 4.611 \times 9810 = 48{,}373.11N/m^2$$

Force in X direction

$$\Sigma F_x = \rho Q(v_{2x} - v_{1x})$$

$$P_1A_1 + F_x - P_2A_2 \cos\theta = \rho Q(v_2 \cos\theta - v_1)$$

$$140 \times 10^3\ (0.196) + F_x - 48.373 \times 10^3 (0.031) \cos 50°$$

$$= 1000\ (0.425)(13.71 \cos 50° - 2.168)$$

$$F_x = -23{,}652.137N$$

$$\therefore F_x = 23{,}652.137N(\leftarrow)$$

Force in Y direction

$$\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$0 + F_y - P_2A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)$$

$$0 + F_y - 48.373 \times 10^3 (0.031) \sin 50° = 1000(0.425)(13.71 \sin 50° - 0)$$

$$F_y = 5612.281N$$

$$\therefore F_y = 5612.281N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{23{,}652.137^2 + 5612.281^2} = 24{,}303.873N$$

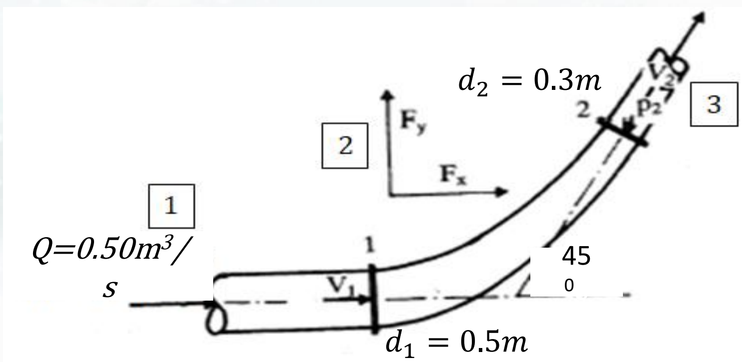
$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{5612.281}{23{,}652.137} = 13.3°$$

$\therefore$ **The resultant force on the pipe bend, $\mathbf{F = 24,303.873N}$,**

and direction of resultant force against the x – axis = 13.3°

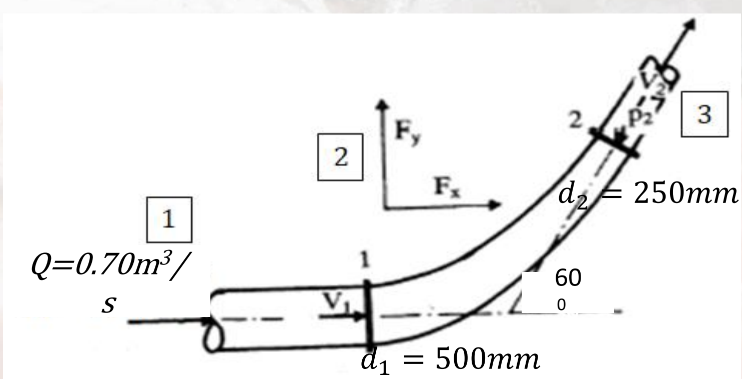
QUESTION 1

A 450 reducing pipe bend taper from 0.5m diameter at inlet to 0.3 m diameter at outlet is flowing a liquid with a specific gravity of 0.9 as shown in figure below. The pressure at inlet is 145 kN/m2 and the flow rate of liquid is 0.5 m3/s. Neglecting any loss in the bend, calculate the magnitude and direction of resultant force exerted by the liquid.



QUESTION 2

A pipe bend was deflected to reduce the pipe diameter from 500mm to 250mm. The deflection of fluid is 600 . The pressure at the bend=160kN/m2 and the flow rate is 0.70m3/s. Based in figure below, Calculate magnitude of resultant force at the bend.



SOLUTION EXERCISE CHAPTER 3

SOLUTION 1

$$A_1 = \frac{\pi(d_1)^2}{4}$$
$$A_1 = \frac{\pi(0.5)^2}{4}$$
$$= 0.196m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4}$$
$$A_2 = \frac{\pi(0.3)^2}{4}$$
$$= 0.071m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.5}{0.196}$$
$$= 2.55m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.5}{0.031}$$
$$= 16.13m/s$$

$$\boldsymbol{Use\ Bernoulli's\ equation, at\ horizontal\ plane, Z_1 = Z_2}$$
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$
$$\left(\frac{145 \times 10^3}{1000 \times 9.81}\right) + \left(\frac{2.55^2}{2 \times 9.81}\right) = \left(\frac{P_2}{1000 \times 9.81}\right) + \left(\frac{16.13^2}{2 \times 9.81}\right)$$
$$15.112 = \left(\frac{P_2}{9810}\right) + 13.26$$
$$P_2 = 1.852 \times 9810 = 18168.12N/m^2$$
$$P_2 = 181.68kN/m^2$$

SOLUTION
EXERCISE CHAPTER 3

SOLUTION 1

Force in X direction

$$\Sigma F_x = \rho Q(v_{2x} - v_{1x})$$

$$P_1A_1 + F_x - P_2A_2\cos\theta = \rho Q(v_2\cos\theta - v_1)$$

$$145\times10^3\,(0.196) + F_x - 181.68\times10^3(0.071)\cos45^\circ$$

$$= 1000\,(0.5)(16.13\cos45^\circ - 2.55)$$

$$F_x = -14871.01N$$

$$\therefore F_x = 14871.01N(\leftarrow)$$

Force in Y direction

$$\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right)$$

$$0 + F_y - P_2A_2\sin\theta = \rho Q(v_2\sin\theta - 0)$$

$$0 + F_y - 181.68\times10^3(0.071)\sin45^\circ = 1000(0.5)(16.13\sin45^\circ - 0)$$

$$F_y = 14823.99N$$

$$\therefore F_y = 14823.99N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(-14871.01)^2 + (14823.99)^2} = 20997.56N$$

$$\emptyset = tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{14823.99}{14871.01} = \mathbf{44.9^\circ}$$

$\therefore$ The resultant force on the pipe bend, F $= 20997.56N$,
and direction of resultant force against the x − axis $= 44.9^\circ$

SOLUTION 2

$$A_1 = \frac{\pi(d_1)^2}{4}$$
$$A_1 = \frac{\pi(0.5)^2}{4}$$
$$= 0.196m^2$$

$$A_2 = \frac{\pi(d_2)^2}{4}$$
$$A_2 = \frac{\pi(0.25)^2}{4}$$
$$= 0.049m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.7}{0.196}$$
$$= 3.57m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.7}{0.049}$$
$$= 14.29m/s$$

$\boldsymbol{Use\ Bernoulli's\ equation, at\ horizontal\ plane, Z_1 = Z_2}$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{P_1}{1000 \times 9.81}\right) + \left(\frac{3.57^2}{2 \times 9.81}\right) = \left(\frac{160 \times 10^3}{1000 \times 9.81}\right) + \left(\frac{14.29^2}{2 \times 9.81}\right)$$

$$\left(\frac{P_1}{9810}\right) + 0.649 = 26.72$$

$$P_2 = 26.071 \times 9810 = 255756.51N/m^2$$
$$P_2 = 255.76kN/m^2$$

SOLUTION
EXERCISE CHAPTER 3

SOLUTION 2

Force in X direction

$$\Sigma \boldsymbol{F_x} = \boldsymbol{\rho Q}(\boldsymbol{v_{2x}} - \boldsymbol{v_{1x}})$$

$$\boldsymbol{P_1 A_1 + F_x - P_2 A_2 \cos\theta = \rho Q(v_2 \cos\theta - v_1)}$$

$$160 \times 10^3\ (0.196) + F_x - 255.76 \times 10^3 (0.049) \cos 60°$$

$$= 1000\ (0.7)(14.29 \cos 60° - 3.57)$$

$$F_x = -22591.38N$$

$$\therefore F_x = -22591.38N(\leftarrow)$$

Force in Y direction

$$\Sigma \boldsymbol{F_y} = \boldsymbol{\rho Q}(\boldsymbol{v_{2y}} - \boldsymbol{v_{1y}})$$

$$\boldsymbol{0 + F_y - P_2 A_2 \sin\theta = \rho Q(v_2 \sin\theta - 0)}$$

$$0 + F_y - 255.76 \times 10^3 (0.049) \sin 60° = 1000(0.7)(14.29 \sin 60° - 0)$$

$$F_y = 19516.09N$$

$$\therefore F_y = 19516.09N(\uparrow)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(-22591.38)^2 + (19516.09)^2} = 29853.78N$$

$$\emptyset = tan^{-1} \frac{F_y}{F_x} = tan^{-1} \frac{19516.09}{22591.38} = \mathbf{40.8°}$$

$\therefore$ The resultant force on the pipe bend, F $= 29853.78N$,
and direction of resultant force against the x − axis $= 40.8°$

Success is the sum of all efforts, repeated Day in and Day out. - Robert Collier

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Practice Problems
- Pipe bends-

1. A 45^0 reducing pipe bend tapers from 600mm diameter at inlet to 300 mm diameter at outlet. At the pressure at inlet is $140kN/m^2$ gauge and the rate of flow of water round the bend is 0.45 m^3/s. At outlet the pressure is 123 kN/m^2 gauge. Neglecting friction , calculate the resultant force exerted by the water on the bend in magnitude and direction. The bend lies in a horizontal plane.

2. A bend in a pipeline gradually reduces from 600mm to 300 mm diameter and deflects the flow of water through an angle of 60^0. At the larger end the gauge pressure is $172kN/m^2$. Assuming there are no frictional losses, determine the magnitude and direction of the resultant force exerted on the bend when the flow is 0.85m3/s.

3. A 400 mm diameter pipe carries water under a head of 30 meters with a velocity of 3.5m/s. If the axis of the pipe turns through 46^0, determine the magnitude and the direction of the resultant force at the bend.

4. A pipe bends was deflected to reduce the pipe diameter from 500 mm to 250mm. The deflection of fluid is 60^0 . The pressure at the bend=$160kN/m^2$. The flow rate is 0.7 m^3/s. Calculate magnitude of resultant force at the bend.

Practice Problems - Pipe bends-

5. A reducing right-angled bend lies in a horizontal plane. Water enters from the west with a velocity of 3m/s and a pressure of 30kPa, and it leaves towards the north. The diameter at the entrance is 500mm and the exit it is 400mm. Neglecting any friction loss, find the magnitude and direction of the resultant force on the bend.

6. The volume of a pipe bend shown in Figure 1 is 250 litres. The flow rate of water in the pipe is 170 litre/s. The pipe diameters at points 1 and 2 are 600 mm and 300 mm respectively. The pressure at points 1 is 200 kPa and the total head loss from point 1 and 2 is 0.3 m. Calculate the x and y components of the force to hold the pipe bend if it is installed in:

i. A vertical plane

ii. A horizontal plane

(Ans: i. F_x = 50.10 kN←, F_y = 13.80 kN, ii. F_x = 49.58 kN←,F_y = 12.25 kN↑

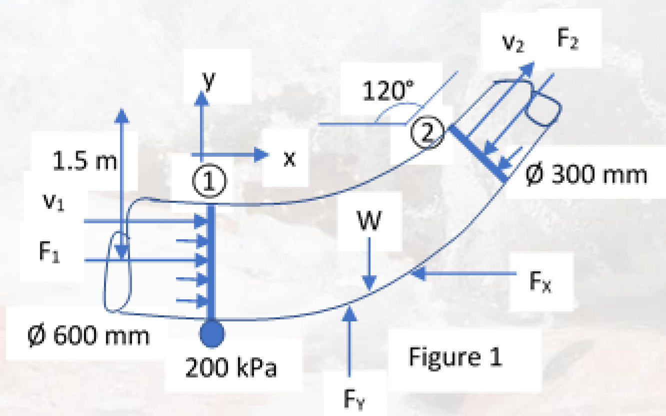

Figure 1

Practice Problems
- Pipe bends-

7. The volume of a pipe bend shown in Figure 2 is 335 litres. The flow rate of water in the pipe is 525 litre/s. The total head loss from point 1 to 2 is 0.4 m. Calculate the x and y components of the force to hold the pipe bend if it is installed in:

i. A vertical plane

ii. A horizontal plane

(Ans: i. F_x = 219.09 kN, F_y = 40.05 kN↓, ii. F_x = 218.59 kN, F_y = 42.48 kN↓)

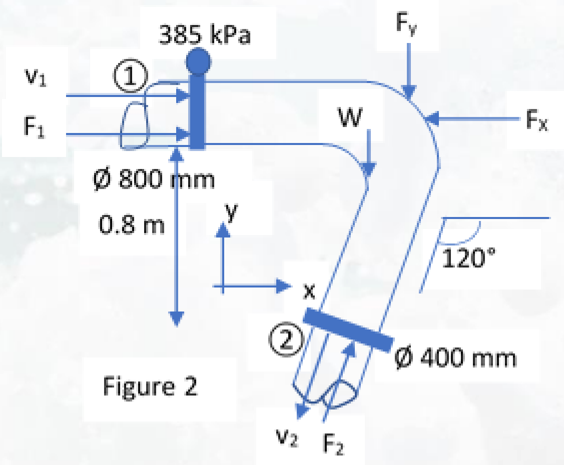

Figure 2

8. The flow rate of water flowing out of an 80 mm diameter nozzle shown in Figure 3 is 60 litre/s. The volume of the water in the pipe bend is 50 litres and the pipe bend is installed in a vertical plane. The head loss from point 1 to 2 is 0.44m. Calculate the x and y force components to hold the pipe bend.

(Ans: F_x = 0.933 kN←, F_y = 0.862kN↓)

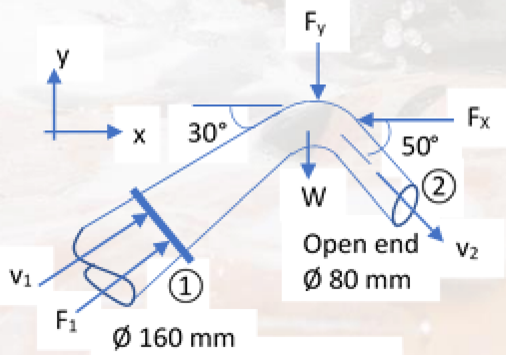

Figure 3

Practice Problems
- Pipe bends-

9. A reducing elbow is used to deflect water flow at a rate of 14kg/s in a horizontal pipe upward 30° while accelerating it (Figure 4). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113cm^2 at the inlet and 7cm^2 at the outlet. The elevation difference between the centres of the outlet and the inlet is 30cm. The weight of the elbow and the water it is considered to be negligible. Determine (a) the gage pressure at the centre of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place.

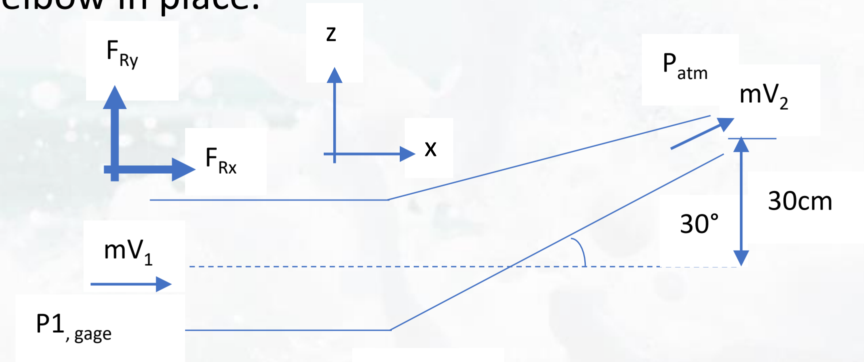

Figure 4

10. A 90° elbow (Figure 5) is used to direct water flow at a rate of 25kg/s in a horizontal pipe upward. The diameter of the entire elbow is 10cm. The elbow discharge water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centres of the exit and the inlet of the elbow is 35cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the centre of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03

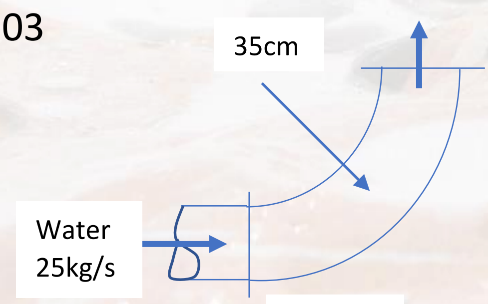

Figure 5

SUMMARY OF THE TOPIC

Principal of Momentum in fluid

$$F = m(V_2 - V_1)$$

$$F = \rho Q(V_2 - V_1)$$

Principal of Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

FORMULA USED

Force act on x-direction

$$\boldsymbol{\Sigma F_x = \rho Q(v_{2x} - v_{1x})}$$
$$\boldsymbol{F_x + F1_x + F2_x = \rho Q(v_{2x} - v_{1x})}$$

Force act on y-direction

$$\boldsymbol{\Sigma F_y = \rho Q(v_{2y} - v_{1y})}$$
$$\boldsymbol{F_y + F1_y + F2_y = \rho Q(v_{2y} - v_{1y})}$$

Resultant force, $\boldsymbol{F = \sqrt{F_x^2 + F_y^2}}$

Direction of resultant,

$$\boldsymbol{\emptyset = tan^{-1}\frac{F_y}{F_x}}$$

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