SULIT

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN MALAYSIA

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI I : 2018/2019

BBM2013 : CALCULUS FOR ENGINEERING TECHNOLOGY

TARIKH : 02 JANUARI 2019
MASA : 9.00 PAGI – 12.00 PETANG

Kertas ini mengandungi **SEMBILAN (9)** halaman bercetak.

Struktur (5 soalan)

Dokumen sokongan yang disertakan : Formula, Kertas Graf

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FIVE (5)** structured questions. Answer **ALL** questions.

ARAHAN:

*Bahagian ini mengandungi **LIMA (5)** soalan berstruktur. Jawab **SEMUA** soalan.*

QUESTION 1

SOALAN 1

CLO1 C1

a) Find the value of the following limits:

Cari nilai bagi had berikut:

i) $\lim_{x \to 0} 7$

$\textit{had}_{x \to 0}\, 7$

[1 mark]
[*1 markah*]

ii) $\lim_{x \to 4} \left(\frac{x^2 - 4x}{x^2 - 16} \right)$

$\textit{had}_{x \to 4} \left(\frac{x^2 - 4x}{x^2 - 16} \right)$

[3 marks]
[*3 markah*]

CLO1 C2

b) Find $\lim_{x \to -2} f(x)$ and hence sketch the graph of $f(x)$ for $-4 \le x \le 0$.

Cari $\textit{had}_{x \to -2} f(x)$ *dan seterusnya lakar graf* $f(x)$ *bagi* $-4 \le x \le 0$.

$$f(x) = \begin{cases} x^2 + 1 & , x < -2 \\ 3x + 1 & , x \ge -2 \end{cases}$$

[6 marks]
[*6 markah*]

CLO1
C3

c) The function f is defined as:

Fungsi f ditakrifkan sebagai:

$$f(x)=\begin{cases} \dfrac{4}{\sqrt{4-x}} & , x<0 \\ \sqrt{2} & , x=0 \\ \dfrac{x}{\sqrt{1+x}-1} & , x>0 \end{cases}$$

i) Show that $\lim_{x \to 0} f(x)$ exists.

Tunjukkan $\underset{x \to 0}{had}\ f(x)$ wujud.

[7 marks]

[*7 markah*]

ii) Determine whether f is continuous at $x=0$.

Tentukan sama ada f selanjar pada $x=0$.

[3 marks]

[*3 markah*]

QUESTION 2

SOALAN 2

CLO2
C1

a) By using the first principle of derivatives, prove that $\frac{dy}{dx}$ for the function $y = 3x^2 + 4$ is $6x$.

Dengan menggunakan terbitan prinsip pertama, buktikan $\frac{dy}{dx}$ *bagi fungsi* $y = 3x^2 + 4$ *adalah* $6x$.

[4 marks]

[*4 markah*]

CLO2
C2

b) Find $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$ for the function $z = \sin(2x + 3y)$.

Cari $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$ *dan* $\frac{\partial^2 z}{\partial y \partial x}$ *bagi fungsi* $z = \sin(2x + 3y)$.

[6 marks]

[*6 markah*]

CLO2
C3

c) Given the implicit function $3x^2 - 2y^2 + 4xy + 13 = 0$, find the gradient of the function at point $(-1, 2)$. Hence, find the equation of the normal line to the curve at point $(-1, 2)$.

Diberi fungsi tersirat $3x^2 - 2y^2 + 4xy + 13 = 0$, *cari kecerunan fungsi tersebut pada titik* $(-1, 2)$. *Seterusnya, dapatkan persamaan bagi garis normal pada lengkung tersebut di titik* $(-1, 2)$.

[10 marks]

[*10 markah*]

QUESTION 3

SOALAN 3

CLO2
C2

a) Let $s(t)=t(t^2-12t+45)$ be the position vector of a particle in meters after t seconds. Calculate the velocity of the particle when its acceleration is zero?

Andaikan $s(t)=t(t^2-12t+45)$ *adalah kedudukan vektor bagi satu zarah dalam meter selepas* t *saat. Kirakan kelajuan zarah tersebut apabila pecutannya adalah sifar ?*

[5 marks]

[*5 markah*]

CLO2
C3

b) Find the coordinates of maximum and minimum points for the curve $y=x^2(x-3)+1$.

Dapatkan koordinat titik maksimum dan minimum bagi lengkung $y=x^2(x-3)+1$.

[10 marks]

[*10 markah*]

CLO2
C4

c) Hydrogen is being inflated into a spherical balloon at a constant rate at $2.5cm^3s^{-1}$. Find the increase rate of radius when the volume of the balloon is $16cm^3$. $\left(V_{sphere}=\frac{4}{3}\pi r^3\right)$.

Hidrogen diisi ke dalam belon berbentuk sfera pada kadar tetap $2.5cm^3s^{-1}$. *Dapatkan kadar perubahan bagi peningkatan jejari apabila isipadu belon adalah* $16cm^3$. $\left(V_{sfera}=\frac{4}{3}\pi r^3\right)$.

[5 marks]

[*5 markah*]

QUESTION 4

SOALAN 4

CLO2
C1

a) Find each of the following integrals:

Cari setiap kamiran bagi setiap fungsi yang berikut:

i) $\int \left(1 + 4x - x^3\right) dx$

[1 mark]
[*1 markah*]

ii) $\int (3x + 6)^2 dx$

[1 mark]
[*1 markah*]

iii) $\int \left(\frac{x^4 + 5x^8}{x^4} \right) dx$

[2 marks]
[*2 markah*]

CLO2
C2

b) Show that $\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \, dx = e - 1$.

Tunjukkan bahawa $\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \, dx = e - 1$.

[6 marks]
[*6 markah*]

CLO2
C3

c) Given an expression $\frac{13-14x-6x^2}{(4-3x)(1+x)^2}$.

Diberi ungkapan $\frac{13-14x-6x^2}{(4-3x)(1+x)^2}$.

i) Determine the partial fraction decomposition of the expression.

Tentukan pengembangan pecahan separa bagi persamaan tersebut.

[5 marks]

[*5 markah*]

ii) Hence, evaluate $\int_0^1 \frac{13-14x-6x^2}{(4-3x)(1+x)^2}\,dx$. Give your answer in 3 decimal places.

Seterusnya, *nilaikan* $\int_0^1 \frac{13-14x-6x^2}{(4-3x)(1+x)^2}\,dx$. *Berikan jawapan anda dalam 3 tempat perpuluhan.*

[5 marks]

[*5 markah*]

QUESTION 5

SOALAN 5

CLO2
C2

a) A curve has a gradient $\frac{dy}{dx} = 6x + 2k$, at a turning point (1,3), where k is a constant. Find:

Satu lengkung mempunyai kecerunan $\frac{dy}{dx} = 6x + 2k$, *pada titik pusingan* (1,3) *dengan k ialah pemalar. Cari:*

i) the value of k.

nilai k.

[2 marks]

[*2markah*]

ii) the coordinates where the curve intersect the y −axis.

koordinat apabila lengkung tersebut bersilang dengan paksi−y .

[3 marks]

[*3 markah*]

CLO2
C3

b) Given a function of a curve as $y = (x-3)(x-9)$.

Diberi fungsi bagi satu lengkung sebagai $y = (x-3)(x-9)$.

i) Determine the minimum point of the curve when $\frac{dy}{dx} = 0$.

Tentukan titik minimum bagi lengkung tersebut apabila $\frac{dy}{dx} = 0$.

[3 marks]

[*3 markah*]

ii) Sketch the graph.

Lakarkan graf.

[3 marks]

[3 *markah*]

iii) Calculate the area bounded by the curve and the x −axis.

Hitung luas rantau yang dibatasi oleh lengkung dan paksi−x.

[4 marks]

[4 *markah*]

CLO2
C4

c) A tennis ball is thrown vertically upwards from the ground level at $4ms^{-1}$. Due to the gravity, the acceleration of the ball is $-9.8ms^{-2}$. Calculate the maximum height reached by the ball.

Sebiji bola tenis dilontar secara mencancang ke atas dari aras tanah pada kadar $4ms^{-1}$. *Disebabkan oleh graviti, pecutan bola itu ialah* $-9.8ms^{-2}$. *Hitung tinggi maksimum yang dapat dicapai oleh bola tersebut.*

[5 marks]

[5 *markah*]

SOALAN TAMAT

FORMULA: BBM2013 CALCULUS FOR ENGINEERING TECHNOLOGY

LIMIT & FUNCTION

$$\lim_{x\to a} c = c$$

$$\lim_{x\to a} x^n = a^n$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

DIFFERENTIATION

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$$

$$\frac{d}{dx}(u\cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

TRIGONOMETRIC IDENTITIES

$$\cos^2 x + \sin^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\cos ec^2 x = 1 + \cot^2 x$$

$$\sin 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

DIFFERENTIATION

$$\frac{d}{dx}(k) = 0;\ k = \text{constant}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\ln|u|) = \frac{1}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\cos ec^2 u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u\tan u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\cos ec\ u) = -\cos ec\ u\cot u\cdot\frac{du}{dx}$$

INTEGRATION

$$\int k\ dx = kx + C;\ k = \text{constant}$$

$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + C;\ n \neq -1$$

$$\int \frac{1}{u}\ du = \frac{\ln|u|}{du/dx} + C$$

$$\int e^u du = \frac{e^u}{du/dx} + C$$

$$\int \sin u\ du = \frac{-\cos u}{du/dx} + C$$

$$\int \cos u\ du = \frac{\sin u}{du/dx} + C$$

$$\int \sec^2 u\ du = \frac{\tan u}{du/dx} + C$$

$$\int \cos ec^2 u\ du = \frac{-\cot u}{du/dx} + C$$

$$\int \sec u\tan u\ du = \frac{\sec u}{du/dx} + C$$

$$\int \cos ec\ u\cot u\ du = \frac{-\cos ecu}{du/dx} + C$$

FORMULA: BBM2013 CALCULUS FOR ENGINEERING TECHNOLOGY

TANGENT LINE EQUATION	NORMAL LINE EQUATION
$y - y_1 = m(x - x_1)$	$y - y_1 = -\frac{1}{m}(x - x_1)$
AREA BOUNDED BY AXIS	**VOLUME REVOLVED AROUND AXIS**
$A = \int_a^b y\,dx$ $A = \int_a^b x\,dy$	$V = \pi\int_a^b y^2\,dx$ $V = \pi\int_a^b x^2\,dy$

INTEGRATION BY PARTS
$uv - \int v\,du$