SULIT

POLITEKNIK MALAYSIA

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI DISEMBER 2015

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 06 APRIL 2016
MASA : 8.30 AM - 10.30AM (2 JAM)

Kertas ini mengandungi **LIMA BELAS (15)** halaman bercetak.
Bahagian A: Struktur (2 soalan)
Bahagian B: Struktur (2 soalan)
Bahagian C: Struktur (2 soalan)
Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

SECTION A : 50 MARKS

BAHAGIAN A : 50 MARKAH

INSTRUCTION:

This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** question from either part A/B/C.

ARAHAN :

*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap satu. Jawab **SATU (1)** soalan dari setiap bahagian, dan **SATU (1)** soalan selebihnya dari mana-mana bahagian samaada A/B/C.*

QUESTION 1

SOALAN 1

CLO1 C1 (a) Find the value of the following functions :

Dapatkan nilai bagi fungsi-fungsi yang berikut :

i. sinh 3 [2 marks] [*2 markah*]

ii. sech (-5) [2 marks] [*2 markah*]

iii. coth 1.5 [2 marks] [*2 markah*]

CLO1
C3

(b) If $y^2 = \frac{x}{2}\tanh(1.5x)$, find the value of y when $x = -8$.

Jika $y^2 = \frac{x}{2}\tanh(1.5x)$, *dapatkan nilai y bila* $x = -8$.

[4 marks]
[*4 markah*]

CLO1
C3

(c) Complete the table below for equation $y = \sinh(2x+1)$. Then sketch the graph in the range given as $-3 \le x \le 2$.

Lengkapkan jadual dibawah bagi persamaan $y = \sinh(2x+1)$. *Seterusnya lakarkan graf pada julat* $-3 \le x \le 2$.

x	-3	-2	-1	0	1	2
y						

[7 marks]
[*7 markah*]

CLO1
C3

(d) Prove that:
Buktikan:

i) $2\sinh x \cosh x = \sinh 2x$

[4 marks]
[*4 markah*]

ii) $\cosh x - \sinh x = e^{-x}$

[4 marks]
[*4 markah*]

QUESTION 2
SOALAN 2

CLO1
C1

(a) Find the value for each of the following by using the definition of hyperbolic functions:
Cari nilai bagi setiap yang berikut dengan menggunakan definisi fungsi hiperbolik:

i) $\text{sech}^{-1}\left(\frac{1}{4}\right) + \text{cosech}^{-1}(-2)$

[4 marks]
[*4 markah*]

ii) $\tanh^{-1}(\cosh 2\pi)$

[3 marks]
[*3 markah*]

iii) $\text{sech}^{-1}\left(\frac{1}{3}\right)$

[3 marks]
[*3 markah*]

iv) $\sinh^{-1} 1.364$

[3 marks]
[*3 markah*]

v) $\tanh^{-1} 0.816$

[2 marks]
[*2 markah*]

CLO 1
C3

(b) Show that $\coth^{-1} t = \frac{1}{2}\ln\frac{t+1}{t-1}$.

Buktikan bahawa $\coth^{-1} t = \frac{1}{2}\ln\frac{t+1}{t-1}$.

[7 marks]
[*7 markah*]

CLO 1
C2

(c) Solve the equation $\sec^{-1} 3y = \frac{\pi}{3}$.

Selesaikan persamaan $\sec^{-1} 3y = \frac{\pi}{3}$.

[3 marks]
[*3 markah*]

SECTION B : 50 MARKS

BAHAGIAN B : 50 MARKAH

INSTRUCTION:

This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** question from either part A/B/C.

ARAHAN :

*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap satu. Jawab **SATU (1)** soalan dari setiap bahagian, dan **SATU (1)** soalan selebihnya dari mana-mana bahagian samaada A/B/C.*

QUESTION 3

SOALAN 3

CLO2
C3

(a) Differentiate each of the following equations with respect to x:

Bezakan setiap persamaan berikut terhadap x *:*

i. $y = \cosh(2-3x^2)$ [3 marks] [*3 markah*]

ii. $y = x^2 \sec^{-1}(2x)$ [4 marks] [*4 markah*]

iii. $y = e^{4x} \tanh^{-1}(x)$ [4 marks] [*4 markah*]

CLO2 C3 (b) Given $z=\sqrt{y}-\sin(xy)+5x^2$. Calculate:

Diberi $z=\sqrt{y}-\sin(xy)+5x^2$. *Kirakan:*

i. $\dfrac{\partial z}{\partial x}$ [3 marks] [*3 markah*]

ii. $\dfrac{\partial z}{\partial y}$ [3 marks] [*3 markah*]

iii. $\dfrac{\partial^2 z}{\partial x^2}$ [3 marks] [*3 markah*]

CLO2 C3 (c) Use implicit differentiation method to determine the derivative for the following functions.

Gunakan kaedah pembezaan tersirat untuk menentukan pembezaan bagi fungsi yang berikut.

$$y+xy=y^2+8x-5$$

[5 marks] [*5 markah*]

QUESTION 4
SOALAN 4

CLO2 C3 (a) Determine the following integrals:

Tentukan setiap kamiran berikut:

i. $\int -6\operatorname{sech}(3x)\tanh(3x)\,dx$ [3 marks] [*3 markah*]

ii. $\int 4x^3 \sinh 5x^4\,dx$ [5 marks] [*5 markah*]

iii. $\int \dfrac{3}{x\sqrt{25x^2-4}}\,dx$ [5 marks] [*5 markah*]

CLO2 C3 (b) Solve the following integral:

Selesaikan kamiran yang berikut:

i. $\int \dfrac{1}{\sqrt{4x^2+16x-65}}\,dx$ [6 marks] [*6 markah*]

ii. $\int xe^{\frac{x}{2}}\,dx$ [6 marks] [*6 markah*]

SECTION C : 50 MARKS

BAHAGIAN C : 50 MARKAH

INSTRUCTION:

This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** question from either part A/B/C.

ARAHAN :

*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap satu. Jawab **SATU (1)** soalan dari setiap bahagian, dan **SATU (1)** soalan selebihnya dari mana-mana bahagian samaada A/B/C.*

QUESTION 5

SOALAN 5

CLO3 C3

(a) Form a differential equation for each of the following functions:

Bentukkan persamaan pembezaan bagi setiap fungsi yang berikut:

i. $y = Ax^2 + Bx$ [7 marks] [7 *markah*]

ii. $y = 4A\cosh 2x - 4B\sinh 2x$ [4 marks] [4 *markah*]

CLO3 C3

(b) Solve the differential equation for $(x^2 + y^2)\frac{dy}{dx} = xy$.

Selesaikan persamaan pembezaan bagi. $(x^2 + y^2)\frac{dy}{dx} = xy$.

[9 marks]

[9 *markah*]

CLO3 C3

(c) Solve the following second order differential equation below.

Selesaikan persamaan pembezaan peringkat kedua di bawah.

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$$

[5 marks]

[5 *markah*]

QUESTION 6

SOALAN 6

CLO3 C3

(a) Solve the following differential equation;

Selesaikan persamaan pembezaan berikut:

i. $\frac{dy}{dx}+2y=e^{2x}$ [4 marks] [*4 markah*]

ii. $\frac{dy}{dx}=\frac{x+y}{2x}$ [7 marks] [*7 markah*]

CLO3 C3

(b) Solve the following second order differential equation:

Selesaikan persamaan pembezaan peringkat kedua berikut:

i. $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=0$ [4 marks] [*4 markah*]

ii. $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ [4 marks] [*4 markah*]

iii. $2\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ [6 marks] [*6 markah*]

SOALAN TAMAT

FORMULA ENGINEERING MATHEMATICS 5

HYPERBOLIC FUNCTIONS	INVERSE HYPERBOLIC FUNCTIONS
$\sinh x = \frac{e^x-e^{-x}}{2}$	$\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right); -\infty < x < \infty$
$\cosh x = \frac{e^x+e^{-x}}{2}$	$\cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right); x \geq 1$
$\tanh x = \frac{e^x-e^{-x}}{e^x+e^{-x}}$	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); \lvert x\rvert < 1$
$\coth x = \frac{e^x+e^{-x}}{e^x-e^{-x}}; x \neq 0$	$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right); \lvert x\rvert > 1$
$\text{sech}\, x = \frac{2}{e^x+e^{-x}}$	$\text{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right); 0 < x \leq 1$
$\text{cosech}\, x = \frac{2}{e^x-e^{-x}}; x \neq 0$	$\text{cosech}^{-1} = \ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right); x \neq 0$

RECIPROCAL TRIGONOMETRIC IDENTITIES	RECIPROCAL HYPERBOLIC IDENTITIES
$\text{cosec}\, x = \frac{1}{\sin x}$	$\text{cosech}\, x = \frac{1}{\sinh x}$
$\sec x = \frac{1}{\cos x}$	$\text{sech}\, x = \frac{1}{\cosh x}$
$\cot x = \frac{1}{\tan x}$	$\coth x = \frac{1}{\tanh x}$

TRIGONOMETRIC IDENTITIES		HYPERBOLIC IDENTITIES	
$\cos^2 x + \sin^2 x = 1$		$\cosh^2 x - \sinh^2 x = 1$	
$1+\tan^2 x$	$= \sec^2 x$	$1-\tanh^2 x$	$= \text{sech}^2 x$
$\cot^2 x + 1$	$= \text{cosec}^2 x$	$\coth^2 x - 1$	$= \text{cosech}^2 x$
$\sin 2x$	$= 2\sin x \cos x$	$\sinh 2x$	$= 2\sinh x \cosh x$
$\cos 2x$	$= \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$	$\cosh 2x$	$= \cosh^2 x + \sinh^2 x$ $= 2\cosh^2 x - 1$ $= 1 + 2\sinh^2 x$
$\tan 2x$	$= \frac{2\tan x}{1-\tan^2 x}$	$\tanh 2x$	$= \frac{2\tanh x}{1+\tanh^2 x}$
$\sin(x \pm y)$	$= \sin x \cos y \pm \cos x \sin y$	$\sinh(x \pm y)$	$= \sinh x \cosh y \pm \cosh x \sinh y$
$\cos(x \pm y)$	$= \cos x \cos y \mp \sin x \sin y$	$\cosh(x \pm y)$	$= \cosh x \cosh y \pm \sinh x \sinh y$
$\tan(x \pm y)$	$= \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tanh(x \pm y)$	$= \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

BASIC OF DIFFERENTIATION	BASIC OF INTEGRATION
$\frac{d}{dx}(k) = 0;\ k = \text{constant}$	$\int k\,du = ku + C;\ k = \text{constant}$
$\frac{d}{dx}(u^n) = nu^{n-1}$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C;\ n \neq -1$
$\frac{d}{dx}(\ln\lvert u\rvert) = \frac{1}{u}\cdot\frac{du}{dx}$	$\int \frac{1}{u}\,du = \frac{\ln\lvert u\rvert}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(e^u) = e^u\cdot\frac{du}{dx}$	$\int e^u\,du = \frac{e^u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	INTEGRATION OF TRIGONOMETRIC FUNCTIONS
$\frac{d}{dx}(\cos u) = -\sin u.\frac{du}{dx}$	$\int \sin u\,du = \frac{-\cos u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sin u) = \cos u.\frac{du}{dx}$	$\int \cos u\,du = \frac{\sin u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\tan u) = \sec^2 u.\frac{du}{dx}$	$\int \sec^2 u\,du = \frac{\tan u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u.\frac{du}{dx}$	$\int \operatorname{cosec}^2 u\,du = \frac{-\cot u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sec u) = \sec u.\tan u.\frac{du}{dx}$	$\int \sec u \tan u\,du = \frac{\sec u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u.\cot u.\frac{du}{dx}$	$\int \operatorname{cosec} u \cot u\,du = \frac{-\operatorname{cosec} u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	INTEGRATION OF HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\cosh u) = \sinh u.\frac{du}{dx}$	$\int \sinh u\,du = \frac{\cosh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sinh u) = \cosh u.\frac{du}{dx}$	$\int \cosh u\,du = \frac{\sinh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u.\frac{du}{dx}$	$\int \operatorname{sech}^2 u\,du = \frac{\tanh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u.\frac{du}{dx}$	$\int \operatorname{cosech}^2 u\,du = \frac{-\coth u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$	$\int \operatorname{sech} u \tanh u\,du = \frac{-\operatorname{sech} u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u.\coth u.\frac{du}{dx}$	$\int \operatorname{cosech} u \coth u\,du = \frac{-\operatorname{cosech} u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS	INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ \lvert u\rvert < 1$	$\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\frac{u}{a} + C,\ \lvert u\rvert < a$
$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ \lvert u\rvert < 1$	$\int -\frac{1}{\sqrt{a^2-u^2}}du = \cos^{-1}\frac{u}{a} + C,\ \lvert u\rvert < a$
$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$	$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2}\frac{du}{dx}$	$\int -\frac{1}{a^2+u^2}du = \frac{1}{a}\cot^{-1}\frac{u}{a} + C$
$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert > 1$	$\int \frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C,\ \lvert u\rvert > a$
$\frac{d}{dx}(\operatorname{cosec}^{-1} u) = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert > 1$	$\int -\frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\operatorname{cosec}^{-1}\frac{u}{a} + C,\ \lvert u\rvert > a$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS	INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$	$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}\frac{u}{a} + C,\ a > 0$
$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert > 1$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}\frac{u}{a} + C,\ u > a$
$\frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2}\frac{du}{dx},\ \lvert u\rvert < 1$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C;\ \lvert u\rvert < a$
$\frac{d}{dx}(\coth^{-1} u) = \frac{1}{1-u^2}\frac{du}{dx}, \lvert u\rvert > 1$	$\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\frac{u}{a} + C;\ \lvert u\rvert > a$
$\frac{d}{dx}(\operatorname{sech}^{-1} u) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx},\ 0 < u < 1$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a} + C$
$\frac{d}{dx}(\operatorname{cosech}^{-1} u) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx},\ u \neq 0$	$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}\operatorname{cosech}^{-1}\frac{u}{a} + C$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations

- Substitution

$y = vx$ and $\frac{dy}{dx} = v + x\frac{dy}{dx}$

Linear Factors (Integrating Factors)

$y \bullet IF = \int Q \bullet IF\,dx$

Where $IF = e^{\int P\,dx}$

Logarithmic

$a = e^{\ln a}$

$a^x = e^{x \ln a}$

$\int a^x\,dx = \frac{a^x}{\ln a} + c$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

1. Real & different roots: $y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots: $y = e^{mx}(A + Bx)$
3. Complex roots: $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$