

SULIT



**BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI**

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI JUN 2016

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 27 OKTOBER 2016

MASA : 8.30 AM - 10.30 AM (2 JAM)

Kertas ini mengandungi **SEMBILAN (9)** halaman bercetak.

Bahagian A: Struktur (2 soalan)

Bahagian B: Struktur (2 soalan)

Bahagian C: Struktur (2 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

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INSTRUCTION:

Answer **ONE (1)** question from each section (A, B and C) and answer **ONE (1)** question from any section that has not been answered.

ARAHAN :

Jawab **SATU (1)** soalan daripada setiap bahagian (A,B dan C) dan jawab **SATU (1)** soalan yang belum dijawab dari mana-mana bahagian.

**SECTION A
BAHAGIAN A****QUESTION 1
SOALAN 1**

CLO 1

C1

- a) Find the value for each of the following by using the definition of hyperbolic functions :
Cari nilai bagi setiap yang berikut dengan menggunakan definisi fungsi hiperbolik :

i) $\cosh(\ln 3)$ [2 marks]

[2 markah]

ii) $\tanh \sqrt{4}$ [2 marks]

[2 markah]

iii) $\sinh x = \frac{5}{12}$, find $\tanh x$ [5 marks]

[5 markah]

iv) $\operatorname{cosech} \frac{5}{12}$ [3 marks]

[3 markah]

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CLO 1

C2

b) If the water wave with length, $L = 40$ m moves with velocity in the depth of water, $d = 5$ m where g is the gravity acceleration $= 9.81 \text{ ms}^{-2}$. Find the velocity, v .

Jika panjang ombak $L = 40$ m bergerak dengan kelajuan kedalaman air, $d = 5$ m di mana g adalah pecutan graviti iaitu $= 9.81 \text{ ms}^{-2}$. Cari nilai kelajuan, v .

$$v = \sqrt{\frac{gL}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)}$$

[5 marks]

[5 markah]

CLO 1

C3

c) Prove that $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

Buktikan bahawa $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

[5 marks]

[5 markah]

CLO 1

C3

d) Determine the principle value of $\cos^{-1}(\cos 50^\circ)$. Sketch the graph.

Dapatkan nilai prinsipal bagi $\cos^{-1}(\cos 50^\circ)$. Lakarkan graf.

[3 marks]

[3 markah]

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QUESTION 2
SOALAN 2

CLO1

C2

(a) Calculate the value for each of the following hyperbolic functions.

Kirakan nilai bagi setiap fungsi hyperbola yang berikut.

i. $\sinh(-5)$

[2 marks]

[2 markah]

ii. $\tanh\left(\frac{1}{3}\right)$

[2 marks]

[2 markah]

iii. $\coth^{-1} 5$

[2 marks]

[2 markah]

iv. $\operatorname{sech}^{-1}(0.2)$

[2 marks]

[2 markah]

CLO1

C3

(b) Prove that :

Buktikan :

$$\cosh x + \sinh x = e^x$$

[4 marks]

[4 markah]

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SECTION B
BAHAGIAN B

QUESTION 3
SOALAN 3

CLO2
C2

a) Find $\frac{dy}{dx}$ for the following functions:

Cari $\frac{dy}{dx}$ bagi fungsi berikut:

i) $y = -2 \sinh^3 3x$

[3 marks]
[3 markah]

ii) $y = \cosh^{-1}(\tan x)$

[4 marks]
[4 markah]

iii) $y = e^{2x} \sinh^{-1} 5x$

[5 marks]
[5 markah]

CLO2
C2

b) Differentiate the following function to x
Bezakan fungsi berikut terhadap x

$$5x^3 + 4x^2y + y^3 = \cosh x$$

[6 marks]
[6 markah]

CLO2
C3

c) If $z = 3x \cos y$, prove that $\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial x \partial y} = -6 \sin y$

Jika $z = 3x \cos y$ buktikan $\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial x \partial y} = -6 \sin y$

[7 marks]
[7 markah]

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CLO1
C2

(c) Sketch a quadrant graph and find the principal value for the following functions.
Lakarkan graf sukuan dan dapatkan nilai prinsipal bagi fungsi-fungsi berikut.

i. $\sin^{-1}(0.95)$

[4 marks]
[4 markah]

ii. $\tan^{-1}(\sqrt{2})$

[4 marks]
[4 markah]

iii. $\cot^{-1}(\sqrt{9})$

[5 marks]
[5 markah]

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QUESTION 4
SOALAN 4CLO2
C3

- (a) Find the integration of the following functions.
Dapatkan pengamiran fungsi dibawah.

i. $\int 7 \tanh\left(3x + \frac{1}{2}\right) dx$

[5 marks]
[5 markah]

ii. $\int \frac{1}{\sqrt{25 - 4x^2}} dx$

[5 marks]
[5 markah]

iii. $\int \frac{1}{\sqrt{9x^2 - 3}} dx$

[5 marks]
[5 markah]

iv. $\int \frac{e^{2x}}{1 + e^{4x}} dx$

[5 marks]
[5 markah]

CLO2
C3

- (b) Integrate $\frac{x}{(x-3)(x+3)}$ to x by using **integration by partial fractions**.

Kamirkan $\frac{x}{(x-3)(x+3)}$ terhadap x dengan menggunakan kamiran pecahan

separa.

[5 marks]
[5 markah]

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SECTION C
BAHAGIAN CQUESTION 5
SOALAN 5CLO3
C2

- (a) Form the differential equation for $y = Ax^2 - Bx + x$.
Bentukkan persamaan pembezaan bagi $y = Ax^2 - Bx + x$

[8 marks]
[8 markah]

CLO3
C3

- (b) Determine the general solution for the following differential equations.
Tentukan penyelesaian am bagi persamaan pembezaan yang berikut.

i. $e^x \frac{dy}{dx} = 4$, given $y = 3$ and $x = 0$.

[5 marks]
[5 markah]

ii. $x \frac{dy}{dx} = 5x^3 + 4$

[5 marks]
[5 markah]

iii. $x \frac{dy}{dx} + y = x^3$

[7 marks]
[7 markah]

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QUESTION 6

SOALAN 6

CLO3
C4

- (a) Solve the differential equation below by using an appropriate method.
Selesaikan persamaan pembezaan berikut dengan menggunakan kaedah yang sesuai.

$$(x^2 + xy) \frac{dy}{dx} = xy - y^2 \quad [10 \text{ marks}]$$

[10 markah]

CLO3
C3

- (b) Solve the following second order differential equation.
Selesaikan persamaan pembezaan peringkat kedua di bawah.

i. $2 \frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 3y = 0$ [4 marks]

[4 markah]

ii. $\frac{d^2 y}{dx^2} + 14 \frac{dy}{dx} + 49y = 0$ [4 marks]

[4 markah]

iii. $\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 7y = 0$ [7 marks]

[7 markah]

SOALAN TAMAT

HYPERBOLIC FUNCTIONS	INVERSE HYPERBOLIC FUNCTIONS
$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}; x \neq 0$ $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$ $\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}; x \neq 0$	$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}); -\infty < x < \infty$ $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}); x \geq 1$ $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right); x < 1$ $\coth^{-1} x = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right); x > 1$ $\operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right); 0 < x \leq 1$ $\operatorname{cosech}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{ x }\right); x \neq 0$
RECIPROCAL TRIGONOMETRIC IDENTITIES	RECIPROCAL HYPERBOLIC IDENTITIES
$\operatorname{cosec} x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$	$\operatorname{cosech} x = \frac{1}{\sinh x}$ $\operatorname{sech} x = \frac{1}{\cosh x}$ $\coth x = \frac{1}{\tanh x}$
TRIGONOMETRIC IDENTITIES	HYPERBOLIC IDENTITIES
$\cos^2 x + \sin^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$ $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\coth^2 x - 1 = \operatorname{cosech}^2 x$ $\sinh 2x = 2 \sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $= 2 \cosh^2 x - 1$ $= 1 + 2 \sinh^2 x$ $\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$ $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$ $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$ $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

BASIC OF DIFFERENTIATION	BASIC OF INTEGRATION
$\frac{d}{dx}(k) = 0; k = \text{constant}$ $\frac{d}{dx}(u^n) = nu^{n-1}$ $\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$ $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$	$\int k du = ku + C; k = \text{constant}$ $\int u^n du = \frac{u^{n+1}}{n+1} + C; n \neq -1$ $\int \frac{1}{u} du = \ln\left \frac{u}{du}\right + C$ $\int e^u du = \frac{e^u}{\left(\frac{du}{dx}\right)} + C$
DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	INTEGRATION OF TRIGONOMETRIC FUNCTIONS
$\frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\sec u) = \sec u \cdot \tan u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u \cdot \cot u \cdot \frac{du}{dx}$	$\int \sin u du = \frac{-\cos u}{\left(\frac{du}{dx}\right)} + C$ $\int \cos u du = \frac{\sin u}{\left(\frac{du}{dx}\right)} + C$ $\int \sec^2 u du = \frac{\tan u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{cosec}^2 u du = \frac{-\cot u}{\left(\frac{du}{dx}\right)} + C$ $\int \sec u \tan u du = \frac{\sec u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{cosec} u \cot u du = \frac{-\operatorname{cosec} u}{\left(\frac{du}{dx}\right)} + C$
DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	INTEGRATION OF HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\cosh u) = \sinh u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\sinh u) = \cosh u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \cdot \tanh u \cdot \frac{du}{dx}$ $\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u \cdot \coth u \cdot \frac{du}{dx}$	$\int \sinh u du = \frac{\cosh u}{\left(\frac{du}{dx}\right)} + C$ $\int \cosh u du = \frac{\sinh u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{sech}^2 u du = \frac{\tanh u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{cosech}^2 u du = \frac{-\coth u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{sech} u \tanh u du = \frac{-\operatorname{sech} u}{\left(\frac{du}{dx}\right)} + C$ $\int \operatorname{cosech} u \coth u du = \frac{-\operatorname{cosech} u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTIONS	INTEGRATION OF INVERSE TRIGONOMETRIC FUNCTION
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, u < 1$ $\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, u < 1$ $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$ $\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$ $\frac{d}{dx}(\sec^{-1} u) = \frac{1}{ u \sqrt{u^2-1}} \frac{du}{dx}, u > 1$ $\frac{d}{dx}(\operatorname{cosec}^{-1} u) = -\frac{1}{ u \sqrt{u^2-1}} \frac{du}{dx}, u > 1$	$\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1} \frac{u}{a} + C, u < a$ $\int -\frac{1}{\sqrt{a^2-u^2}} du = \cos^{-1} \frac{u}{a} + C, u < a$ $\int \frac{1}{a^2+u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$ $\int -\frac{1}{a^2+u^2} du = \frac{1}{a} \cot^{-1} \frac{u}{a} + C$ $\int \frac{1}{ u \sqrt{u^2-a^2}} du = \frac{1}{a} \sec^{-1} \frac{u}{a} + C, u > a$ $\int -\frac{1}{ u \sqrt{u^2-a^2}} du = \frac{1}{a} \operatorname{cosec}^{-1} \frac{u}{a} + C, u > a$
DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS	INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$ $\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}, u > 1$ $\frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}, u < 1$ $\frac{d}{dx}(\coth^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}, u > 1$ $\frac{d}{dx}(\operatorname{sech}^{-1} u) = -\frac{1}{u\sqrt{1-u^2}} \frac{du}{dx}, 0 < u < 1$ $\frac{d}{dx}(\operatorname{cosech}^{-1} u) = -\frac{1}{ u \sqrt{1+u^2}} \frac{du}{dx}, u \neq 0$	$\int \frac{1}{\sqrt{a^2+u^2}} du = \sinh^{-1} \frac{u}{a} + C, a > 0$ $\int \frac{1}{\sqrt{u^2-a^2}} du = \cosh^{-1} \frac{u}{a} + C, u > a$ $\int \frac{1}{a^2-u^2} du = \frac{1}{a} \tanh^{-1} \frac{u}{a} + C; u < a$ $\int \frac{1}{u^2-a^2} du = \frac{1}{a} \coth^{-1} \frac{u}{a} + C; u > a$ $\int \frac{1}{u\sqrt{a^2-u^2}} du = -\frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a} + C$ $\int \frac{1}{u\sqrt{a^2+u^2}} du = -\frac{1}{a} \operatorname{cosech}^{-1} \frac{u}{a} + C$
INTERGRALS INVOLVING QUADRATIC EXPRESSION	
<p>Completing the square</p> $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$	

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations

- Substitution

$$y = vx \quad \text{and} \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Linear Factors (Integrating Factors)

$$y \cdot IF = \int Q \cdot IF \, dx$$

$$\text{Where } IF = e^{\int P \, dx}$$

Logarithmic

$$a = e^{\ln a}$$

$$a^x = e^{x \ln a}$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + c$$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$

1. Real & different roots:

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

2. Real & equal roots:

$$y = e^{m x} (A + Bx)$$

3. Complex roots:

$$y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$$